Ramon Llull: From the Ars Magna to Artificial Intelligence

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(Eds.)

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Contents

Preface

The lay philosopher and theologian Ramon Llull (1232–1316), born in Mallorca, is undoubtedly a prominent figure within European thought. However, the exact position he occupies within the cultural horizons of his period, on the one hand, and the intellectual legacy he bequeaths to the present day, on the other, are issues often immersed in controversy. This situation derives, in part, from the protean multiplicity of his writings, manifested by an impressive variety of forms, styles and subject matter, Llull having composed some 280 works in both Catalan and Latin (as well as reputedly in Arabic).

Running throughout his immense oeuvre, nevertheless, is a leitmotif that enables one to arrive at an overall, if not unitary, view, that leitmotif being the *Ars lulliana* or Lullian Art: a philosophico-theological system that makes use of common basic concepts from the three monotheistic religions of its day, subjecting them to discussion with a view to convincing Muslims (and Jews) via rational argument of the truth of the Christian mysteries of faith. By revising his Art and extending it to all fields of human knowledge, Ramon Llull succeeded in creating a universal science, based on the algebraic notation of its basic concepts and their combination by means of mechanical figures. As a matter of fact, Llull not only presented his system to the masters of the University of Paris as well as to the Pope, but he undertook several missionary trips to North Africa in order to put his *Ars* into practice disputing with Muslims in the market place in Bejaia and other cities.

From a more abstract point of view, Llull's combinatorial Art can be described as a process of elementary analysis and of reconstruction. On the one hand, it resolves the historical religions into their most primitive elements; on the other, it represents these elements by letters (from B to K), in order to recombine these letters and the elements of the different religions that they designate until, through these combinations, a vision of the world is reached that is as consistent as possible: this will correspond to truth. Undoubtedly, this process which Llull applied to all kinds of question —not just religious controversies— is a key ingredient of modern thought. One only has to think of Gottfried Wilhelm

Leibniz's *characteristica universalis*: thus, in his *Dissertatio de arte combinatoria*, in 1666, the young Leibniz, clearly inspired by Llull, had already outlined the project of a reconstruction of the whole of reality based on a definite number of basic notions. Leibniz criticizes the basic notions of the Lullian "alphabet" as too limited and proposes another alternative and broader alphabet. In contradistinction to Llull, Leibniz does not represent these basic notions with letters but rather uses numbers. Thus, the basic notion of "space" is represented by the number 2, the basic notion of "between" by the number 3, and the basic notion of "the whole" by the number 10. Consequently, according to Leibniz, a complex concept such as, for instance, "interval" can be formulated as 2.3.10, that is, "space between the whole". Leibniz was convinced that in this way all questions could be reduced to mathematical problems and that, in order to solve any problem, we only have to set about calculating. This is the meaning of Leibniz's famous "Calculemus!"

It is through Leibniz that Llull's influence also became decisive for more recent developments such as formal logic, as developed by Gottlob Frege in the late 19$^{\text{th}}$ century. According to Frege, Leibniz's *characteristica*, in its later evolution, limited itself to different fields, such as arithmetic, geometry, chemistry and so on, but did not become universal as Leibniz, in fact, had wished. This is why Frege, in his famous *Begriffsschrift* from 1879, intended to create an elementary language that would unify the different formal languages which, after Leibniz, had been established in the different natural sciences. This language developed into the formal logic that until now has dominated the philosophical discourse and which was an important step in the journey towards the creation of computing languages. What characterizes this kind of logic is its formal notation, using variables and symbols to represent the different logical propositions and operations. Based on this notation, Frege developed the so-called logical calculus. Although the language reached by this formal logic differs from that of the Art, Llull can be considered as the forerunner of this project, insofar as in his thought one can already find the idea of an elementary language that follows logical rules and uses variables while operating with the principle of substitution of these variables.

However, it is also necessary to stress the differences between Llull's Art and the evolution of modern logic and Artificial Intelligence. Particularly since the early 20^{th} century, with the so-called Vienna Circle, to which thinkers such as Rudolf Carnap belonged, the project of an elementary language has been increasingly linked to the idea of the elimination of all metaphysical expressions of ordinary language. Thus, the project of an elementary language, conceived from the logical-mathematical paradigm, has become programmatically anti-metaphysical. In contrast, Llull's Art was clearly conceived as an explicitly metaphysical elementary language.

In addition to what has been said so far, Llull's contributions to the history of ideas not only embrace formal logic and argumentation theory, but also social choice. Recently rediscovered texts show Llull to have anticipated important work on election theory by several centuries. As a matter of fact, salient features of the Borda and Condorcet voting mechanisms were discussed by him already in the 13^{th} century, i.e. almost 500 years earlier than the authors usually credited with developing these concepts. The present publication, which gathers contributions from experts in Artificial Intelligence, computer science, economics, logic and philosophy, tries to critically assess the pioneering work of Llull in some of these areas, pointing to both its historical presuppositions and its systematic potential.

Although the vindication of the pioneering work of Ramon Llull is the true motivation for this book, the excuse for its publication comes from the celebration in Barcelona of the 22^{nd} International Joint Conference on Artificial Intelligence (IJCAI) in July 2011. It is the first time that this conference has come to Catalonia and the Catalan AI research community wants to offer this book as a present to the attendees from all over the world. Artificial Intelligence has been a very active research area in Catalonia since its initial steps in the late 70s growing into more than twenty-five research groups nowadays. The creation of the Artificial Intelligence Research Institute (IIIA-CSIC) and the Catalan AI association (ACIA) in 1994 were two important landmarks in this growth. Since then, the IIIA has become an internationally recognized research institute and ACIA became the backbone of the AI research in Catalonia through its annual conferences. On behalf of the Catalan AI research community, we hope this book boosts interest in further study on the impact of Ramon Llull's ideas through the centuries and up to current AI research.

Alexander Fidora and Carles Sierra
Barcelona, 20 June 2011

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1 What Was Llull Up To?

Anthony Bonner

In the histories of logic and of computer science which mention precursors and pioneers, we often find mention of Leibniz, the great 17th-century mathematician and philosopher, who was one of the first to try to build a mechanical calculator, and who tried to formulate a *Mathesis universalis*, a sort of scientific language which would permit any two disputants to settle their differences merely by taking pencil and paper and saying, "Let us calculate". But then we find that Leibniz got certain important ideas from a character called Ramon Llull who lived in the 13th century, who came from a place called Mallorca, and who spent his life trying to convert Muslims and Jews. As if this weren't peculiar enough, when he has appeared in modern treatises, it has usually been as the typical romantic genius or in his case, medieval mystic who wasn't quite right in the head. Even a man as sensible as Martin Gardner [8] calls him quixotic and paranoid! So what Llull was up to is a question that indeed needs a bit of clarification.

Perhaps the best way to begin is by trying to situate him in his time. He was born on Majorca around 1232, only two or three years after the King of Aragon and Catalonia had recovered the island from the Muslims. This meant that Llull grew up in an island that was still strongly multicultural. Muslims continued to represent perhaps a third of the population, and Jews, although a much smaller minority, were an important economic and cultural force on the island. So when at the age of thirty he was converted from a profligate youth and he decided to devote his life to the service of the Church, it seemed only logical to do so by trying to convert these "infidels", as they were then called. And he decided to do this in three ways: (1) to develop a system that his adversaries would find difficult to refute (which is what we'll see in a moment), and to try to persuade them of the truth of Christianity instead of just trying to refute their own doctrines, as his predecessors had done;[1] (2) to be willing to risk his life in proselytizing among Muslims and Jews (he in fact made three trips to North Africa); and (3) to try to persuade Kings and Popes of the need for setting up language schools for missionaries, for which purpose he travelled many times throughout France and Italy. He lived to 83 or 84, an incredible age when the average life-span was around 40, dying in 1316.[2]

Now this situation has presented historians with two serious paradoxes. The first is that, if he was principally interested in converting Muslims and Jews, what could this possibly have to do with his being a pioneer of computer science? It would seem doubtful that 13th-century unbelievers would have wanted to listen to arguments that looked forward to Bill Gates, or that modern computer scientists would deem their profession useful for the persuasion of Muslims and Jews of the truths of Christianity. The second paradox is that the system Llull thought up doesn't look like anything his contemporaries were using, nor can it be considered really acceptable to modern logic. This second paradox has caused enormous problems for historians of logic. Those equipped with a knowledge of medieval logic who try to tackle Llull are disagreably suprised to find him discussing either not at all or passing over very superficially the topics they feel he *should* be discussing, and which they know his contemporaries *were* discussing. Those who try to tackle it from the point of view of modern formal logic are understandably put off by his basing his system on an extreme Platonic realism, and thus making it depend primarily on meaning rather than form. Llull himself was aware of these problems, and carefully tried to explain that his system was *neither* logic nor metaphysics. But that only helps us to understand what it *isn't*; what it is is something I will try in very broad outline to explain now. But before doing so, I would ask you to suspend, for the moment at least, your highly trained and normally indispensable sense of disbelief, and only start apply-

[1] The tactic was psychologically important. Instead of forcing his adversaries to justify their own faith, he gave them the opportunity to "falsify", as Popper would say, Christianity.

[2] And not dying martyred, as pious legends would have it. He is buried in Palma, in the Church of Sant Francesc.

ing it again when we've seen a bit of the inside of the edifice Llull constructed, because if not, we'll never get past the front door.

The first thing we have to face is the problem of his trying to persuade unbelievers. From the outset Llull realized that previous attempts had failed because people had based their arguments on sacred texts. Christians argued positively trying to explain the truths of the Bible, or negatively trying to point out the errors in the Qur'an or in the Talmud. Such discussions, however, invariably became bogged down in arguments as to which texts were acceptable to whom, and how to interpret them. Since it was clearly impossible for opposing sides to agree on these points, such discussions never got anywhere. Participants invariably left them with a feeling of having tried unsuccessfully to walk uphill in sand.

So Llull decided to try something completely abstracted from the specific beliefs of any one religion, based only on whatever beliefs or areas of knowledge they had in common. All three religions, for instance, were monotheistic, and none of them could deny that this one God of theirs had a series of positive attributes: goodness, greatness, eternity, etc. They also shared a common heritage of Greek science which taught them about the earth at the center of a universe with seven planets rotating around it, and that this earth of ours was composed of four elements, fire, earth, air and water. And the framework in which all three philosophized about the world was that of Aristotle. Finally, all could agree more or less about what constituted virtues and vices.

What Llull then set out to do was to show how one could combine these theological, scientific and moral components to produce arguments that at least couldn't be rejected outright by his opponents. It was furthermore clear that if he was going to set up an *Ars combinatoria*, as later generations called it, its components would have to be finite in number and clearly defined. Since they were like the premisses of his arguments, everybody had to be quite clear as to what they were and how they functioned. Saying that people retained visual images better than words, he decided to present his system graphically. This he did in two stages: the first version of his system had twelve or more figures, and he finally had to jettison it in the face of contemporary complaints about its being too complicated.[3] The second version in which the figures were reduced to four is the one for which he was chiefly known in the 16th and 17th centuries, and which we will present here. The final version of this second system found expression in two works: the *Ars generalis ultima*,[4] along with a much shortened introductory version of the same, the *Ars brevis*,[5] which follows the longer one chapter by chapter, but in outline form. These works begin with an "Alphabet" giving the meaning of nine letters, in which he says, "B signifies goodness, difference, whether?, God, justice, and avarice. C signifies...", and so on, all of which can best be set out in a table (see Table on page 9).[6]

He then sets out the components of the first column in his First Figure, or Figure A (p. 10).

Notice first of all, as always with Llull, the letters don't represent variables, but constants. Here they're connected by lines to show that in the Divinity these attributes are mutually convertible. That is to say that God's goodness is great, God's greatness is good, etc. This, in turn was one of Llull's definitions of God, because in the created world, as we all know too well, people's goodness is not always great, nor their greatness particularly good, etc. Now such a system of vertices connected by lines is what, as mathematicians, you will of course recognize as a graph. This might seem to be of purely anecdotal interest, but as we shall see in a moment, the relational nature of Llull's system is fundamental to his idea of an *Ars combinatoria*.

The components of the second column are set out in a Second Figure, or Figure T.[7] (See Figure on page 11).

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[3]This first version of the Art (of which we'll get a glimpse at the end of this paper) was logically quite different from the second, and included two interesting attempts to establish a new notation, one in the *Ars notatoria* (see [11]), and the other in the *Introductoria Artis demonstrativae* printed in *MOG III*.

[4]The best edition is that of *ROL XIV*.

[5]The Latin text is in *ROL XII*, and an English translation in [12] and [13].

[6]The reader mustn't be disturbed by the lack of the letter J, which didn't exist as a separate letter in Llull's time. The Middle Ages used I/J as well as U/V interchangeably; they weren't differentiated till the Renaissance.

[7]It received the letter T because in the previous version of the Art the alphabet used not just nine but all the letters of the alphabet, and the position of this figure

	Fig. A	Fig. T	Questions and Rules	Subjects	Virtues	Vices
B	goodness	difference	whether?	God	justice	avarice
C	greatness	concordance	what?	angel	prudence	gluttony
D	eternity*	contrariety	of what?	heaven	fortitude	lust
E	power	beginning	why?	man	temperance	pride
F	wisdom	middle	how much?	imaginative	faith	accidie
G	will	end	of what kind?	sensitive	hope	envy
H	virtue	majority	when?	vegetative	charity	ire
I	truth	equality	where?	elementative	patience	lying
K	glory	minority	how? and with what?	instrumentative	pity	inconstancy

* or duration.

The alphabet of the *Ars brevis*

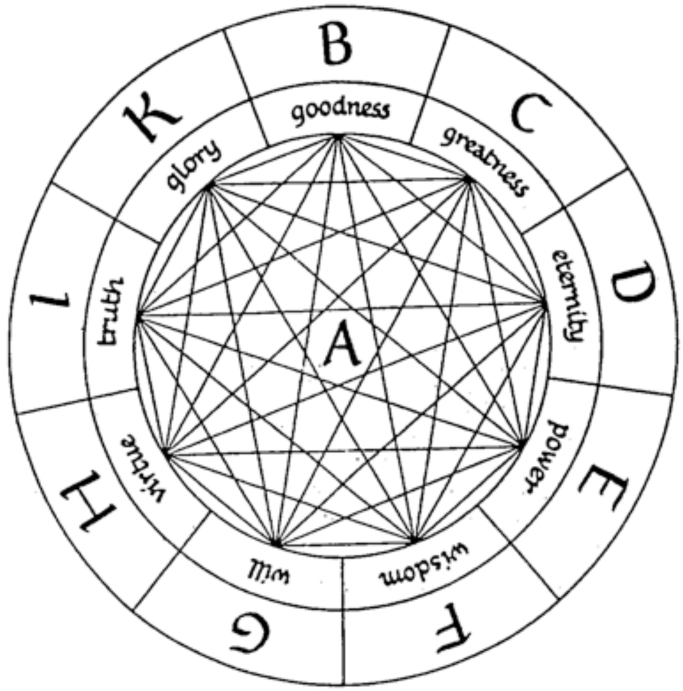


Figure A

Here we have a series of relational principles related among themselves in three groups of three, hence the triangular graphs. The first triangle has difference, concordance, and contrariety; the second beginning, middle, and end; and the third majority, equality, and minority. The concentric circles between the triangles and the outer letters show the areas in which these relations can be applied. For example, with the concept of difference, notice how it can be applied to sensual and sensual, sensual and intellectual, etc. "Sensual" here means perceivable by the senses, and Llull explains in the *Ars brevis*, that: "There is a difference between sen-

followed that of a Figure S. Even though the intervening letters and figures disappeared from his system, Llull, perhaps not to confuse users of the earlier system adapting to his "update", continued referring to it as Figure T.

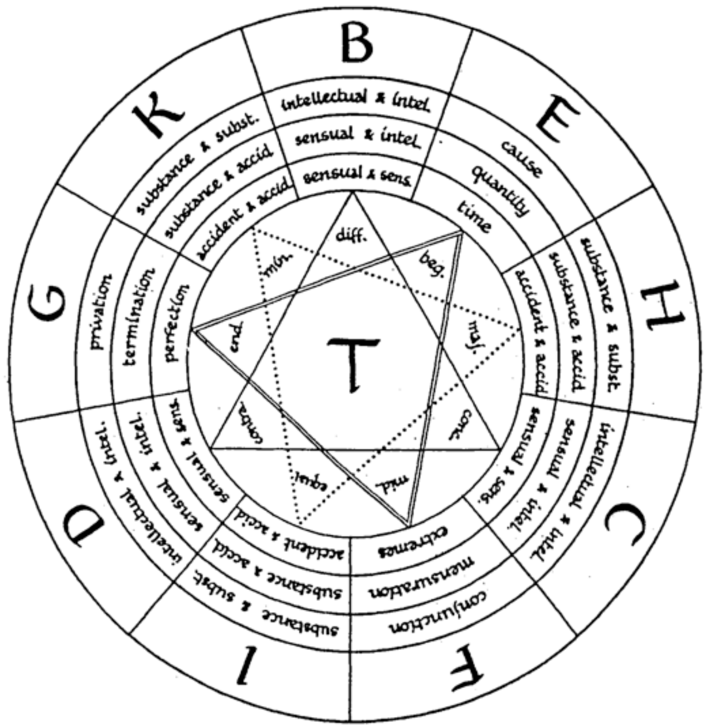

B
intellectual & intel.
sensual & intel.
sensual & sens.
E
cause
quantity
time
H
substance & subst.
substance & accid.
accident & accid.
C
intellectual & intel.
sensual & intel.
sensual & sens.
F
conjunction
mensuration
extremes
I
substance & subst.
substance & accid
accident & accid
D
intellectual & intel.
sensual & intel.
sensual & sens.
G
privation
termination
perfection
K
substance & subst.
substance & accid
accident & accid
diff.
beg.
maj.
conc.
mid.
equal
contra.
end
min.
T

Figure T

sual and sensual, as for instance between a stone and a tree. There is also a difference between the sensual and the intellectual, as for instance between body and soul. And there is furthermore a difference between intellectual and intellectual, as between soul and God".

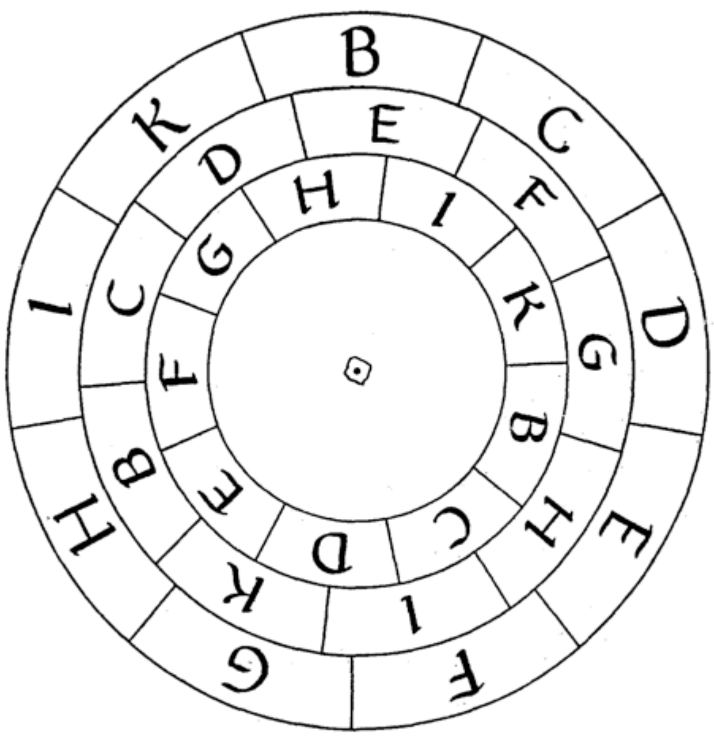
The Third Figure combines the first two: Here Llull explains

BC	CD	DE	EF	FG	GH	HI	IK
BD	CE	DF	EG	FH	GI	HK	
BE	CF	DG	EH	FI	GK		
BF	CG	DH	EI	FK			
BG	CH	DI	EK				
BH	CI	DK					
BI	CK						
BK							

Third Figure

that B C, for instance, implies four concepts: goodness and greatness (from Figure A), and difference and concordance (from Figure T), permitting us to analyze a phrase such as "Goodness has great difference and concordance" in terms of its applicability in the areas of sensual/sensual, sensual/intellectual, and intellectual/intellectual. It furthermore, as he points out, permits us to do this systematically throughout the entire alphabet. This is important, because one of the ways in which Llull conceived his Art as "general" was precisely in its capacity to explore *all* the possible combinations of its components. Now as mathematicians, you will recognize this figure as a half matrix, and you will also see that, in relation to the graph of the First Figure, it is an adjacency matrix. Because such a matrix is symmetrical (in Llull's case this means he makes no distinction between B-C and C-B), he saw no reason to reproduce the other half; and because his graph admits no loops (that is, omits relations such as B-B), he could also omit the principal diagonal.

If the Third Figure explores all possible binary combinations, the Fourth Figure does the same for ternary combinations.



Fourth Figure

In medieval manuscripts, the outside circle is normally drawn on the page, and the two inner ones are separate pieces of parchment or paper held in place on top of it by a little piece of string, permitting them to rotate in relation to each other and to the larger circle. In a moment we'll see how he uses these ternary relations, but before going on let me quote a book on logic for computer applications [17]. Its authors say that one of the things lacking in classical Aristotelian logic was the notion of a relation with many arguments. His predicate relations P(x) were unary, and what he missed was the basic building-block character of binary relations R(x,y) and ternary relations S(x,y,z). This shows that imbedded in what Künzel and Cornelius [10] have called the "hardware" of Llull's system we already have a full panoply of binary and ternary relations.

Binary relations are worked out more extensively in a section he calls "The Evacuation of the Third Figure". For the "compartment", as he calls it, of B C, he not only uses "goodness" and "greatness" from the First Figure, and "difference" and "concordance" from the Second Figure, but also the first two questions of the third column of the alphabet, those also corresponding to the letters B C, which are "whether?" and "what?". This means that for the combination of "goodness" and "greatness" one has three possibilities, a statement and two questions:

- Goodness is great.
- Whether goodness is great.
- What is great goodness?

and so on for "goodness" and "difference", "goodness" and "concordance", for a total of 12 propositions and 24 questions.

Ternary relations are worked out in a Table based on the Fourth Figure (see next page).

The one we show here is the shortened form from the *Ars brevis*; instead of 7 columns, the full form of the *Ars generalis ultima* has 84! Here the letter T acts as a separator: the letters that precede it in any one compartment are from Figure A whereas those that follow it are from Figure T. In addition the first letter can act as an indicator of what question from the third column of the alphabet should be considered. So, for instance, the ninth entry of the first column, B T B D, could be translated as "Whether goodness contains in itself difference and contrariety".

BCD	CDE	DEF	EFG	FGH	GHI	HIK
BCTB	CDTC	DETD	EFTE	FGTF	GHTG	HITH
BCTC	CDTD	DETE	EFTF	FGTG	GHTH	HITI
BCTD	CDTE	DETF	EFTG	FGTH	GHTI	HITK
BDTB	CETC	DFTD	EGTE	FHTF	GITG	HKTH
BDTC	CETD	DFTE	EGTF	FHTG	GITH	HKTI
BDTD	CETE	DFTF	EGTG	FHTH	GITI	HKTK
BTBC	CTCD	DTDE	ETEF	FTFG	GTGH	HTHI
BTBD	CTCE	DTDF	ETEG	FTFH	GTGI	HTHK
BTCD	CTDE	DTEF	ETFG	FTGH	GTHI	HTIK
CDTB	DETC	EFTD	FGTE	GHTF	HITG	IKTH
CDTC	DETD	EFTE	FGTF	GHTG	HITH	IKTI
CDTD	DETE	EFTF	FGTG	GHTH	HITI	IKTK
CTBC	DTCD	ETDE	FTEF	GTFG	HTGH	ITHI
CTBD	DTCE	ETDF	FTEG	GTFH	HTGI	ITHK
CTCD	DTDE	ETEF	FTFG	GTGH	HTHI	ITIK
DTBC	ETCD	FTDE	GTEF	HTFG	ITGH	KTHI
DTBD	ETCE	FTDF	GTEG	HTFH	ITGI	KTHK
DTCD	ETDE	FTEF	GTFG	HTGH	ITHI	KTIK
TBCD	TCDE	TDEF	TEFG	TFGH	TGHI	THIK

Table

So much for the bare mechanics of the Art. Beyond that Llull wanders even farther from the path of modern logic by basing his Art not on the form of his propositions, but on the meaning of their premisses. It is therefore much more intensional than extensional. How this side of his Art functions can perhaps best be explained by making a brief excursion into Lullian definitions, and into the questions and rules.

Now these definitions of his were based on how he felt the world functioned. He proposed, in fact, a vision of reality which was as novel as the system he built. He said that nothing whatever (and of course for him, much less God) was inactive. Nothing just sat there *being* itself; it also *did* whatever its nature called upon it to do. He often used the analogy of fire which wasn't only a thing in itself, but also was active in the production of heat. So also was goodness not only a thing in itself, as, for instance, an essential attribute of God, but it also produced goodness, and this in two ways: interiorly making His greatness, etc. good, and exteriorly creating the world's goodness (or lack of it where evil was concerned). Here again he frequently used the analogy of fire, which in itself creates a flame and heat, and exteriorly, as he said, causes the water in a pot to boil. Moreover, anything active has to have a point of departure (in the case of the thing that produces good, he called it "bonificative"), an object which it affects (the "bonifiable"), and the act itself going from one to the other (that is, which "bonifies"). And it wasn't only God's attributes that were active in this way; every rung of the scale of being was similarly articulated with the three correlatives (as he called them) of action. At the bottom of the ladder, fire had its "ignificative", "ignifiable", and "ignifies", and in the middle, the human mind had "intellective", "intelligible" or "understandible", and "understanding". The world was thus for him a vast dynamic web of ternary relations working both individually or interiorly, as I said before, and exteriorly one upon the other. It was this web of relations that was implied by his definitions. For example, "goodness" the first component of Figure A, he defined interiorly as "that thing by reason of which good does good". But notice how the exterior definition of the second component, "greatness", as "that by reason of which goodness, duration, etc. are great", implies that even goodness could also be defined similarly in terms of the other components of Figure A. So these definitions, which to some commentators have seemed simply tautological, in fact imply a dynamic reality articulated in a large web of interrelations.

Now this definitional doctrine turns up under one of the questions of the third column of the Alphabet of the Art. Not under the first question of "whether?" which inquires into the possibility of a thing existing, but under the second which asks "what" a thing is. This question (or rule, as Llull also calls it) is divided into four species. In the *Ars brevis* Llull uses the example of the intellect instead of goodness to illustrate how it works, saying that "The first [species] is definitional, as when one asks, What is the intellect? To which one must reply that it is that power whose function it is to understand". Notice how this is identical with that of "goodness" as being "that thing by reason of which good does good". The second species goes further and asks, "What does the intellect have coessentially in itself? To which one must reply that it has its correlatives, that is to say, intellective, intelligible, and understanding, without which it could not exist, and would, moreover, be idle and lack nature, purpose, and repose". This refers, of course, to the ternary dynamic structure we already mentioned. We're also by now familiar with the third species, which is when one asks, "What is the intellect in something other than itself? To which one must reply that it is good when understanding in goodness, great when understanding in greatness, etc.". Here we are with the equivalent of "greatness" as being "that by reason of which goodness, duration, etc. are great" which we saw before. The rest of the questions and rules continue in the same vein, carefully distinguishing the different ways in which one can formulate questions such as "of what?" which inquires about material qualities, "why?" which asks about formal causes, "how much?" concerning quantity, "of what kind?" concerning quality, and so on.

So when Llull starts combining elements of the first two figures to answer questions or make proofs, he carefully shores up his arguments with the appropriate definitions and rules. I won't show you how this works in practice, because it would involve delving into too many minutiae of his explanations. I would just like to make a few general remarks. The first to answer a doubt that has probably occurred to you: how can Llull prove anything useful if, as I said before, he limits himself to such divine attributes such as goodness, greatness, etc., which seem hopelessly vague and general in nature? The answer is that in the first place he occasionally lets one see how definitions can be more widely applicable than they might seem. In the above definition of the intellect, for instance, when he says "it is good when understanding in goodness, great when understanding in greatness, etc.", he adds "and [it is] grammatical in grammar, logical in logic, rhetorical in rhetoric, etc.", so right away we are applying these concepts to other fields. Secondly, notice how in the Alphabet, the fourth column of "Subjects" is a ladder of being in which "everything that exists is implied, and there is nothing that exists outside it", as Llull says in the *Ars generalis ultima* (IX, I). The ninth chapter of that work offers a detailed study of each rung in terms of the 18 principles of Figures A and T, and in terms of the 9 rules. The last rung of Instrumentative includes the moral instruments of the virtues and vices which appear in the last two columns of the alphabet, and which, in his more popular works, Llull uses as important tools of persuasion. In yet another adventure into outside material, Llull presents a chapter on "Application" which gives definitions of what he calls the "Hundred Forms" to which the mechanisms of the Art can also be applied. Here he includes every subject imaginable: physical, conceptual, geometrical, cosmological, social, etc. Lest you think we're still operating in a sort of misty area of vague generalities, let me offer the counter-example of Form no. 96 on Navigation in the *Ars generalis ultima*, which in fact consists of a little five-page manual with worked examples of how to find your position at sea!

Your chief objection that this continual reference to the real world (in the Platonic sense that Llull understood it) on which the Art is firmly based, places it at an opposite pole from any kind of formal logic is undeniable. As I said before, however, Llull was aware of this point, and was at pains to make clear that his Art was neither logic nor metaphysics. My feeling, however, is that the Platonic basis of his system is not without historical or conceptual interest; we must remember Leibniz's comment that if someone could reduce Plato's thought to a system, he would render humanity a great service.[8] Secondly, Llull's invention of an *ars combinatoria* as the only possible way of dealing with interrelationships of Platonic forms, was to have a considerable impact in the Renaissance, and would, as my colleague Ton Sales will explain, have a decisive influence on Leibniz. In one sense, however, Llull's system was more abstract and more amenable to analysis by modern mathematical methods. This was in his attempt to systemize not only totally but even semi-mechanically its all-embracing relational nature. This is, of course, what we mean when we say that he developed an *ars combinatoria*. His use of graphs, along with their alternate representation as matrices, to display the relational structure of his system shows a certain understanding of the general nature of the problem. But there is

[8] Quoted in [18].

another aspect of his system which also has curious modern parallels.

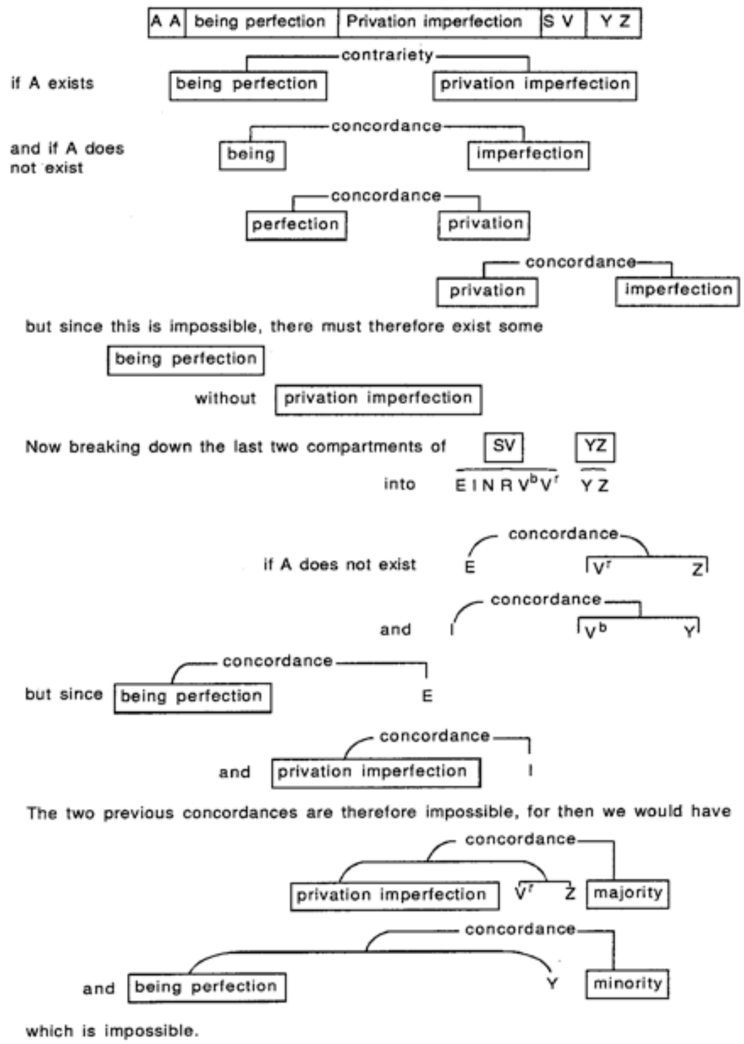
The first period of the Art, which we haven't touched on here, very frequently developed its arguments or proofs by pairwise comparison of concepts. Let me briefly show one such proof from a central work of that period, the *Ars demonstrativa*. Here he always starts his proofs with a series of concepts within what he calls a compartment (or *camera*), as you can see in the Figure on page 20.

The four words not presented by letter symbols come from a Figure X, which disappeared from the later version of the Art, and as you can see, they represent opposites, "privation" being a synonym of "non-being". Notice also the words "contrariety" and "concordance" written above the compartments, which you will recognize as coming from Figure T.

Now A stands for God, and it is double because he is exploring two hypotheses, a positive and negative. The positive one presents no problem: if God exists, there exists a perfect being contrary to privation (or non-being) and imperfection. If, however, God does not exist, then all being has some imperfection, and the only thing that's perfect is non-being or privation, which of course accords with imperfection. Since the concordance of perfection and imperfection is clearly contradictory, the existence of God has been proved by *reductio ad impossibile*. I won't explain the second half of the proof, except to say that it functions similarly.

As you can see, the technique of beginning with a hypothesis and working down a branching structure to a confirmation or refutation, bears a certain resemblance to the tableaux methods of Gentzen, Beth and Smullyan. Notice furthermore how it works by a series of pairwise comparisons.

Which brings me to a further curious piece of evidence recently brought to light by two English scholars. In the social sciences, the modern deductive theory of voting was initiated in the 1950s by Arrow (1951) and Black (1958), with techniques of paired comparisons which in graph theory are called "tournaments". Now the usual history of voting theory says that they were preceded by two Frenchmen, Borda and Condorcet in the


A A | being perfection | Privation imperfection | S V | Y Z
if A exists
contrariety
being perfection
privation imperfection
and if A does not exist
concordance
being
imperfection
concordance
perfection
privation
concordance
privation
imperfection
but since this is impossible, there must therefore exist some
being perfection
without
privation imperfection
Now breaking down the last two compartments of
SV
YZ
into
E I N R V^b V^r
Y Z
if A does not exist
concordance
E
V^r
Z
and
concordance
I
V^b
Y
but since
concordance
being perfection
E
and
concordance
privation imperfection
I
The two previous concordances are therefore impossible, for then we would have
concordance
privation imperfection
V^r
Z
majority
and
concordance
being perfection
Y
minority
which is impossible.

18^{th} century, whose discoveries were forgotten and repeated from scratch by Lewis Carroll, whose work was again utterly neglected. What [14, 15] have shown is that Condorcet and Borda were preceded by half a millenium by Ramon Llull, "who made one of the first systematic contributions to the deductive theory of voting", and this with slightly varying systems presented in two different works. One is aptly called the *Ars electionis*, but the other one is, of all things, embedded in the novel *Blaquerna*, where Llull uses it to explain how nuns should elect their abbess!

What's significant about this, it seems to me, is not so much Llull as the neglected genius, but rather as a thinker with enough breadth of vision to see in his discoveries a generality greater than the initial uses for which they were intended. To a professional mathematician of the late 20^{th} century the connections between "tournaments", graph theory and combinatorics is obvious, but that a 13^{th}-century Majorcan missionary should have seen the connection is, I think interesting.

I would like to end on a more personal note, or what in the scientific community could fall under the euphemistic heading of a call for papers. On the negative side, we have shown that Llull's Art was not a formal logic, but the positive side is unusual and still in many ways in need of explanation. It was a highly structured system, to the point of being semi-mechanical. And the more one deals with it, the more consistent and interesting it seems to become. Lastly, its structure was relational and combinatorial, thus mirroring a world which Llull saw as primarily relational. Might these factors not make it possible to program at least part of the Art in a relational language such as Prolog? And if so, might this not clarify to us, that is, by putting it into modern terms, the functioning of this 13^{th}-century computer? The basic problem, as I see it, is that here we have inherited an ancient computer made of parchment and ink, but along the way the manual got lost. We have many of the materials to make a new one, and if you ask, well, what use would it be, I would answer what a professor from New York University answered some years ago. He was an arachnologist, and when a reporter asked him what good spiders were, he replied, "Spiders are damned interesting, that's what good spiders are".

Abbreviations:
MOG = *Raymundi Lulli Opera omnia*, Ivo Salzinger, editor, 8 vols., Mainz, 1721-1742 (reprint Frankfurt, 1965)
ROL = *Raimundi Lulli Opera Latina*, F. Stegmüller et al., editors, 33 vols. published so far, Palma de Mallorca/Turnhout, Belgium, 1959ff.

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POSTLUDE Almost fifteen years after giving this talk at a computer science congress in Majorca, there is one error which I would like to correct. The description of Figure A on p. 8, with the corresponding illustration on p. 10, represents a confusion between that figure in the two stages of Llull's Art, as does the description of its components as "divine attributes" (which should be simply "principles") on p. 17. Since it is not an error which affects the arguments presented, I have decided it was simpler to leave it rather than trying to go into the complicated and distracting details of the differences between the two versions of the figure. The interested reader can consult my *The Art and Logic of Ramon Llull. A User's Guide* (Leiden - Boston: Brill, 2007), pp. 125-8, for a clarification. For an updating of the bibliography on the theory of voting, the reader can consult the article by Josep M. Colomer in this volume.

Bibliography

[1] E. W. Beth. *Formal Methods; An introduction to symbolic logic and to the study of effective operations in arithmetic and logic*. Dordrecht, D. Reidel, 1962.

[2] A. Bonner. *Ramon Llull: relació, acció, combinatòria i lògica moderna*. Studia Lulliana 34, 51–74, 1994.

[3] E. Colomer. *Ramon Llull, precursor de la informàtica*. Boletín CITEMA 81, 7–44, 1979.

[4] E. Colomer. *De Ramon Llull a la moderna informàtica?*. Estudios Lulianos 23, 113–135, 1979.

[5] N. Deo. *Graph Theory, with Applications to Engineering and Computer Science*. Englewood Cliffs, N.J., Prentice-Hall, 1974.

[6] J. O. Fleckenstein. *Leibniz's algorithmic interpretation of Lullus' art*. Organon (Institut d'Histoire de la science et de la technique auprès de l'Académie Polonaise des Sciences) 4, 171–180, 1967.

[7] L. R. Foulds. *Graph Theory Applications*. New York/Berlin, Springer-Verlag, 1992.

[8] M. Gardner. *The Ars Magna of Ramon Lull*. Logic Machines and Diagrams, Chicago, University of Chicago Press, 1982.

[9] G. Gentzen. *The Collected Papers of Gerhard Gentzen*. Amsterdam, North-Holland, 1969.

[10] H. Cornelius and W. Künzel. *Die Ars Generalis Ultima des Raymundus Lullus. Studien zu einem geheimen Ursprung der Computertheorie*. Berlin: Advanced Studies in Modern Philosophy and Computer Science, 1991.

[11] R. Llull. *Ars notatoria*. Madrid, J. Gayà, editor, CITEMA, 1978.

[12] R. Llull. *Selected Works of Ramon Llull (1232-1316)*. A. Bonner, editor, 2 vols. Princeton, N.J., Princeton University Press, 1985.

[13] R. Llull. *Doctor Illuminatus. A Ramon Llull Reader*. A. Bonner and E. Bonner, editors, Princeton, N.J., Princeton University Press, 1993.

[14] I. McLean and J. London. *The Borda and Condorcet Principles: Three Medieval Applications*. Social Choice and Welfare 7, 99–108, 1990.

[15] I. McLean and J. London. *Ramon Lull and the Theory of Voting*. Studia Lulliana 32, 21–37, 1992.

[16] A. H. Maróstica. *Ars combinatoria and Time: Llull, Leibniz and Peirce*. Studia Lulliana 32, 105–134, 1992.

[17] A. Nerode and R. A. Shore. *Logic for Applications*. New York/Berlin, Springer-Verlag, 1993.

[18] B. Orio. *Leibniz y la tradición neoplatónica. Estado actual de la cuestión*. Revista de Filosofía 7, 493–517, 1994.

[19] R.M. Smullyan. *First Order Logic*. Berlin, Springer-Verlag, 1968.

2 Llull as Computer Scientist, or *Why Llull Was One of Us*

Ton Sales

2.1 Introduction

Something unusual has happened to Ramon Llull, the devote thinker from Majorca. He has been at the same time derided and hailed as a philosopher. He has been instrumental in creating our foundational insights as computer scientists and logicians, yet he occupies a very minor place in the histories of Philosophy, Mathematics or Logic. He was one of the first philosophers to claim a logical basis for religious belief, yet he has been considered a source of alchemy, cabbalistics and mysticism. He is considered a conceited eccentric fool and —just read Martin Gardner's 1958 piece— a maze of confused thinking, but such indictment hardly squares with the undeniable fact that he had foresights which anticipated developments 700 years in the future. So, what is the truth? And what is the man?

That Llull is really a marginal sidepiece in the history of Western Philosophy is clear —as it was to him. And, because he resented it, he innovated, and tried to convince the Parisian intellectuals that his innovative ideas had merit —to no avail. He was not understood at his first Sorbonne appearance in 1289. His combinatorics were definitely not the method to use for logical analysis (causal chaining was). When he came back in 1309–11 with a more accessible system he was greeted with a flurry of sympathy rather than real acceptation. Some found in him a firm advocate of basing faith solely on logic, and all understanding on reason (against the revelationists and the mystically-inclined). But after his death the sympathy faded out, a victim of the Inquisition and the dominican-franciscan 14$^{\text{th}}$-century struggle. In an ironic twist, Llull, who had always put logic before faith, and had done this by propounding innovative ideas, became a thinker derided by the first science pioneers (Bacon or Descartes, who had a lot to thank him for), while he became the hero of alchemists, cabbalists and general mystics (thanks to being attributed authorship of esoteric apocrypha). The usual charge today (for Gardner, as it was for Descartes), that his thinking was actually confused, is not the whole reason for the misrepresentation of Llull's thought: confusion between religious faith, ethical motives, apologetics and natural explanations was the rule rather than the exception in medieval philosophy. As for his own equivocations, it is a well-known fact that all innovators muddle through their own discoveries —the full reach of which they do not usually grasp— and even extrapolate wildly from them. Llull's peculiar innovations, strange as they seemed at the time, sound familiar to the modern ear. Here we list some of the more typically Lullian ones.

2.2 Some basic Computer Science concepts

2.2.1 The idea of a Calculus

That logical reasoning is, in some sense, computation —or, more properly, that it can be formalized and validated by controllable means— is now an accepted idea, clearly explained in the writings of 1920's logicians (Hilbert or Herbrand, to name two) and actually mechanized in the 1960's. But the notion was advanced in the 17^{th} century by Hobbes, who wrote in 1655 that "reasoning is but reckoning", and by Leibniz, who thought in 1658 (and wrote in 1666) that, in the future, philosophers would settle their disputes as accountants do, just by taking pens and calculators (*abaci*) and proclaiming "let us compute!". Leibniz explicitly stated that this was Llull's dream made true. It really was. Llull had anticipated this in 1274 by noting that, to convert Muslims (a current worry), public disputations were fruitless (the ones attempted in the 1260s ended circularly, with nobody convinced), so one had to find a mechanism to prove and generate truths in such a way that, once everyone agreed on the assumptions, the objectivity of the procedure would force all to accept the conclusions. The elaboration of such a mechanism took his lifetime's efforts. Though Leibniz explained the idea in concise and appealing terms, Llull himself could have subscribed to his admirer's formulation, stated 400 years later. Moreover, Llull's "mechanism" was not a merely abstract procedure; it was supported by truly "mechanical" means, his rotating concentric rings. Now easily dismissed as banal toys, they were the first such devices on offer. From this elementary mechanism, and by simple mechanical manipulations, a whole heuristic followed and a deductive chain of truths was combinatorially generated, to be later explored and validated.

2.2.2 The idea of an Alphabet of Thought

When George Boole tried in 1847 and 1854 to find out and formalize the "laws of thought", he basically conceived "thought" as a set of algebraically-expressed concept manipulations. Llull in 1274 also did, but unlike Boole he felt he not only needed a set of allowable manipulations (combinations) but also a finite set of elementary truths to begin with. These he called "dignities" (a plural to translate the greek "axioms") or "absolute principles", nine in number, plus 45 additional basic concepts (in groups of nine) which he called "relative principles" (including consistency or contradiction), "rules" (including quantity or modality), "subjects", "virtues" and "vices". He added to them basic manipulation rules (essentially a relational calculus) and a validation procedure (basically, expanding possible combinations and following them until either two concepts reinforced themselves — thus lending credence to the conclusion— or else a contradiction appeared —which meant that the hypothesized conclusion had to be negated). As Boole later, Llull firmly believed that human thought (logical reasoning) was amenable to symbolic treatment, unified procedures and objective follow-up and control.

2.2.3 The idea of a Method

Not every philosopher in Llull's time felt compelled to delineate clearly a general method, follow it strictly and pretend this was universal. This rather modern concept comprises Hilbert's "effective procedure" idea, or Turing's machine algorithm, but nothing of the sort existed before Descartes suggested the existence and applicability of a universal "method" (1637). Though he did in no way acknowledge his Lullian debt, Descartes —who knew Llull's work well— did for philosophy what the Majorcan had suggested for logical inquiry in general: establish a set of rules, if possible permanent and universal, and follow them strictly.

2.2.4 The idea of Logical Analysis

Llull's idea was to analyze basic concepts by associating them one to each other and see what happened. This, to him, was tantamount to penetrating the inner workings of God and nature, and so to understanding the world better (and giving an effective, objective account of it). If faith (or even mystic revelation) was reached in the process, then Llull's ultimate design purpose was accomplished, that of founding faith on reason, and of justifying beliefs through logical analysis. The originality here was that this was done in practice by mechanically executing an iterated expansion of a given set of initial beliefs (a core or "compendium" of truths) until, should the case arrive, a contradiction obtained. By postulating such a procedure Llull was in fact anticipating the modern (1955) idea of semantic tableaux. (More of this later.)

2.2.5 The idea of Heuristics and Deduction

Llull was interested in *finding* out new truths as well as *proving* —i.e. being able to convince anyone of— old ones. The last part is subsumed in what we ordinarily call *deduction*. The first one ("finding") is somewhat amazing, though. Modern science has systematically eschewed the analysis of why we discover or invent things. This has been attributed to imagination, to brilliancy or even to serendipity, but nobody has tried to explain, and in no way control, how the heuristic process develops. One reason for that is that we can only "control" it *a posteriori*, once the idea has arisen: we can then verify whether the predictions it makes turn out to be true or not. This simple reason was clearly stated by Popper in the 1950s during his dispute with Carnap on the idea of "inductive logic". Besides this, there is nothing we have today to find out new ideas except some magnificent insights into the creative process by Polya and others, and a very short collection of hints or rules-of-thumb for systematic exploration, be this Fred Zwicky's "morphological" method (an exhaustive combinatorial association) or its more modern computer-oriented counterparts in Artificial Intelligence (complete with more or less *ad hoc* techniques we AI-ers pretentiously call "heuristics"). What catches the eye most is that a thorough-modern method as Zwicky's, with its exploratory and pairing algorithms and tables, is strikingly similar, even in its outside appearance and paraphernalia, to the visual tools of Llull's. Needless to say, Heuristics as a science (if it ever was one) is nowadays in the same sorry state in which Llull found it.

2.2.6 The idea of Generative Systems

Perhaps the most striking of Llull's anticipations was the idea of having a *finite* set of *rules* as well as a finite set of truths —"basic concepts", *axioms* or whatever you call it—, so that you can then generate from them a (presumably infinite) set of *derived* truths. Nowadays we would describe the idea more simply, and say that Llull had just come across the idea of a *generative* system. In linguistics such a finitistic device is called a *grammar* (a set of rules to manipulate strings from an alphabet beginning with some initial axioms) and the generated strings are the *language*. In Computer Science the device is called a *machine* and what is being generated is the set of output configurations in a tape. As is well known today, the same mechanism can run backwards: the same grammar that is capable of *generating* a language is also capable of *accepting* or *recognizing* its strings as belonging to it. Or the same machine which computes the batch of acceptable results is also capable of recognizing a correct calculation. (That those two dual processes are slightly asymmetric in computational terms is a corollary of Gödel's first incompleteness theorem and should not bother us here.) Llull was the first to notice this reversible duality: in his terms, the same system that he proposed to derive new truths from a reduced set (an abridged "compendium" of them) and that he called "truth-finding procedure" ("art de trobar veritat" in Catalan or "ars inveniendi" in Latin) and that in Logic we now call simply *inference* (or "forward chaining") had a dual quality and could be executed in reverse, so that we then have a recognizing or accepting system he called "truth-proving procedure" ("art de demostrar", "ars demonstrandi") and we name simply *proof* (or "backward chaining" or "goal-oriented search" in AI). Thus, to Llull, if one were confronted with proving some specific statement, one would have to invent no new system: the one that allowed the user to explore new truths would suffice to certify the intended truth, the certification procedure itself being the proof.

2.2.7 The idea of a Graph

Llull connected his "basic concepts" with lines, and prescribed that the lines had to be followed to combine the concepts and derive the consequences. This was new. Not now, though; we have a name for the device Llull invented: we call it a *graph*. The two amazing things about this are, first, that Llull gave a dual isomorphic variety of it: he compiled the graph's information in the form of a two-entry table (just what we term the *adjacency matrix* of the graph) and, second, that Llull's graphs were not meant as mere concept-structuring or taxonomic (tree-structured concepts were available since late Roman times) but were conceived rather as a present-day's "semantic network" and intended to be "followed", i.e. dynamically executed as though it were a truly fact-finding "program" or a decision tree (as in AI) in a decision procedure.

2.2.8 The idea of Tableaux

The truth-deriving procedures Llull suggested were mainly two. One proceeded in the positive sense: concepts were combined (following the directing graphs) and, if mutually reinforcing, they proved the conclusion "by analogy". The negative dual was that at some point the concepts that were being currently manipulated turned out to be mutually inconsistent (contradictory); that meant that the initial postulated truth was automatically disproved, thereby proving the contrary. This is the first appearance in the literature of something not unlike Beth's 1955 *semantic tableaux* (or Popper's 1959 refutational ideas in science). It is, however, a mere 13th-century anticipation of present-day developments, which —

unlike all the other insights mentioned— have not been directly influenced by Llull's ideas or by Leibniz's rendition of them.

2.2.9 The idea of Conceptual Nets

As previously mentioned, Llull's graphs were neither static taxonomical trees nor concept-structuring illustrations but an actual net of links that allowed the user to explore in a combinatorial fashion the relations that existed among the currently manipulated concepts. Thus they were a prefiguration of modern so called conceptual graphs and semantic networks. They were meant to be not so static or self-structuring but rather they presupposed a dynamic interpretation: to know well the concepts meant — to Llull— to follow their associations and explore their consequences. (The inherent dynamic ontology such a vision gave of things was enthusiastically received by philosophical and scientific innovators like the influential 15^{th}-century Nicholas of Cusa.)

2.2.10 The idea of Diagrams

The universally known Venn diagrams (actually Euler's) also trace their historical lineage up to Llull. He was the first to show how to represent graphically his "concepts" by circles and link them by superposing and intersecting them (though his aim was not to show whether they had an intersection but to demonstrate that they had a more or less strong affinity). (He also linked the terms of a syllogism with a triangle, in what was later called "pons asinorum".) Llull's circle drawings of concepts became a learning aid in J. L. Vives' hands in the 16^{th} century, and were perfected in the 17^{th} by Sturm or Leibniz —who created a whole (unpublished) logical notation out of them— and finally, yes, Euler (in the 1760s). (Now we call them, improperly, Venn's diagrams.)

2.3 The origins

An interesting thread for historians to follow is how and where Llull got his own pioneering notions. For some of them we have a hint. Thus, the idea of starting from a finite set of rules to develop a whole system has a remote ancestor in Euclides and the

Alexandrian Greeks and a more recent and innovative version in Al-Khwarizmi's "algebra" work. This book, translated into Latin shortly before Llull's time, created a sensation with its novel idea of rule-directed manipulations and prescribed "algorithms" (a concept and a word derived from the Muslim mathematician). On the other hand, Llull's idea of a comprehensive method to encompass such rules and develop concepts was probably a formal extension of a now-forgotten integral part of medieval education: the complex set of elaborated techniques for reminding and structuring things in human memory in a printless age (actually, Llull's method was developed in this sense by Petrus Ramus in the 16th century and was then an inspiration for Bacon and Descartes). As for the mechanical devices (the rotating disks), we now know that similar "question-answering disks" were on sale in the 1260s or 1270s in Algeria (as tools for divination), and that Llull could well know them before his 1274 formulation. (Llull's disks met an unexpected use in cryptography, when Leon Battista Alberti, Leonardo's mentor, first used them for coding, and we can still recognize them in the rotors of the WW II German Enigma machine, a distant echo of Llull's disks.)

2.4 The consequences

Llull is not a forgotten anticipator, nor a mere precursor. Llull's work, which had to pay the unexpected toll of being augmented with all kinds of apocrypha that were falsely attributed to him, was well known and appreciated by many influential thinkers of the Renaissance and after. He had a strong influence on —but no explicit recognition by— such people as Montaigne, Pascal, Descartes or Newton (who had Llull in his library, a fact that put him on a par with his arch-enemy Leibniz). Giordano Bruno and Leibniz not only got the influence but were not afraid to acknowledge it. Leibniz is our most direct connection with Llull. By looking for a universal notation and a universal way of acquiring and developing knowledge more or less inspired by the methods of Mathematics (his *mathesis universalis*), he avidly absorbed Llull, critically adapted him and proposed an objective and mechanical way of founding Logic and rational inquiry. In this he failed, after leaving a string of unpublished notes (which included an algebra of thought and a graph formalism), and only some 150 years later could his blocked program be unleashed by Boole's insights. But other Leibnizian ideas went ahead, notably his push for concept decomposition and analysis which had two unexpected derivations: (1) the analysis of minute quantities (the "infinitesimals", on whose development and rights his discussion with Newton turned dismally bitter) and (2) the actual construction in the 1670s of a calculating machine (the first practical multiplier, which prompted an unanticipated reflection by Leibniz on the idoneity of the *binary* system for calculating). Leibniz's thoughtful 1666 'Dissertatio de arte combinatoria' is not only good and interesting reading for today's logicians and mathematicians. It is the best criticism and homage that Llull has ever received: by recognizing his merits and adapting his ideas to the modern needs of Science, Leibniz did all to include Llull in our scientific heritage, and did us a favor in the process.

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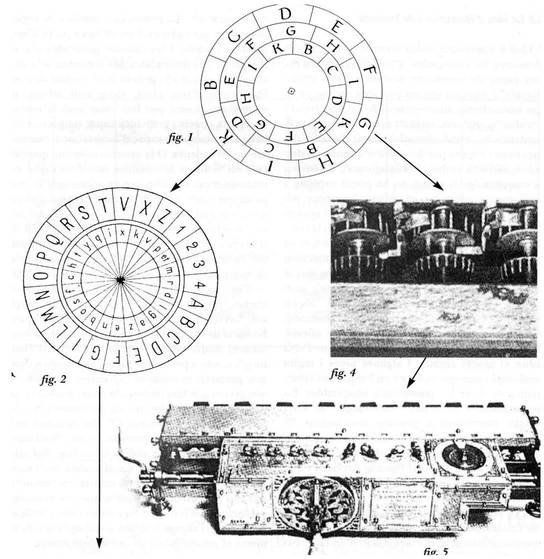
POSTLUDE: The posterity of Llull's rings

In the following pages I include a series of figures that illustrate the pioneering work of Ramon Llull.



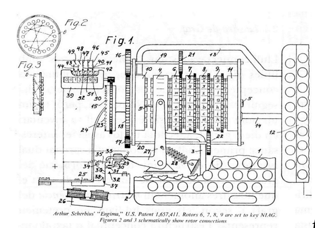


fig. 3

fig. 1: The first calculator in history (with apologies to the Antichytera computer).
fig. 2: Leon Battista Alberti's "cyphering disc" (16^{th} c.) (Alberti, a devoted Llullist, was Leonardo's master and role model).
fig. 3: The last cryptographic avatar of Llull's ring: the Enigma machine of WW II.
fig. 4: Llull's rings now truly "mechanic" (here inside Pascal's 1640 calculator). Transition from Alberti to Schickart and Pascal was made possible by Leonardo da Vinci's design for a calculator (see fig. 6).
fig. 5: The rings in Leibniz's calculator (1674), still decimal, could have been "binary".

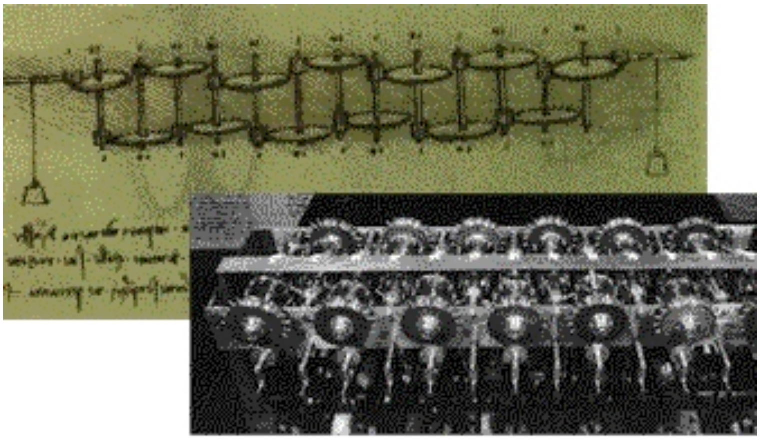
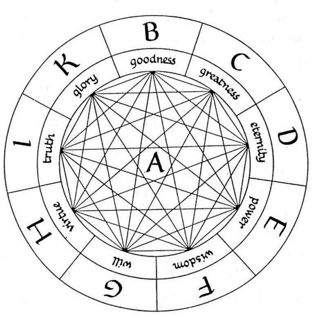
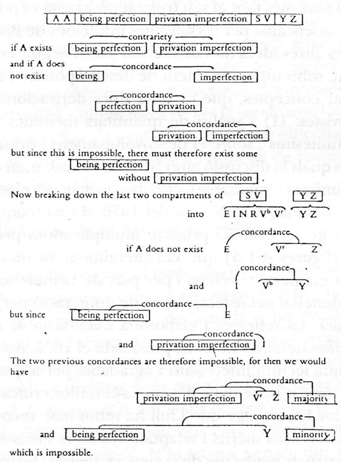


fig. 6: Leonardo's proto-calculator: original drawing and modern implementation.



BC	CD	DE	EF	FG	GH	HI	IK
BD	CE	DF	EG	FH	GI	HK	
BE	CF	DG	EH	FI	GK		
BF	CG	DH	EI	FK			
BG	CH	DI	EK				
BH	CI	DK					
BI	CK						
BK							

fig. 7: (Left) The graph (here a totally connected one) . . .
fig. 8: (Right) . . . And the matrix (the "adjacency matrix", conveniently triangular because of the graph's symmetry).
(with apologies to Tony Bonner, from whom these two figures are plagiarized).



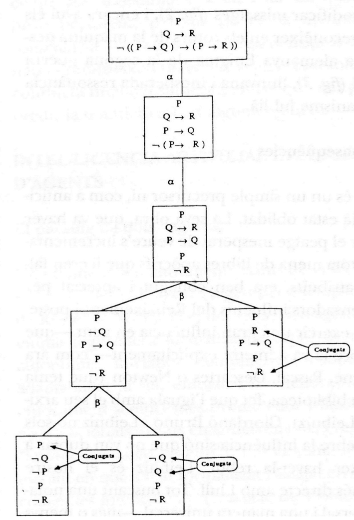


fig. 9: (Left) How to find a contradiction (="impossible!"), (Bonner's version —see page 20 for a better quality image).
fig. 10: (Right) Semantic tableau (the modern version of the "impossible").

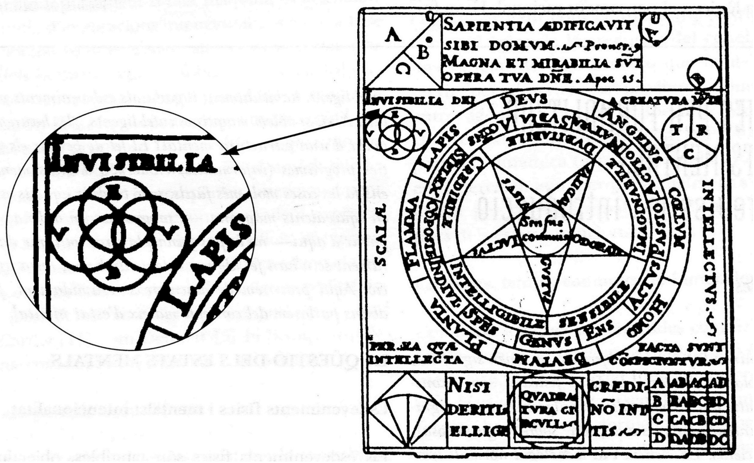


fig. 11: The first "Venn" (here in a 15th c. German version, following Llull's suggestions).

3 Ramon Llull's Contributions to Computer Science

John N. Crossley

Ramon Llull was born on the island of Mallorca and he was a Catalan. He writes a very simple Latin, which is usually easy to understand and translate. In addition he learnt Arabic, believing that to convert Muslims he should understand their language. In 1958 Martin Gardner, [15], was the first modern to draw attention to Llull's contributions to logic machines. Llull's contributions to what has become mathematical logic and computer science have been noted by various authors over the years, but their assessments have oscillated between brief dismissal and adulation. Thus Prantl ends a long exposition with a sharp dismissal of Llull:

> That the whole art of Lull is worthless now needs no more specific evidence. ([31], vol. 3, pp. 145–177, at p. 177.)

On the other hand Sowa ([38, pp. 5–6]) graciously acknowledges one of Llull's contributions to computer science in that Llull's rotating discs generated large numbers of combinations which could then be tested. Other writers, such as Bonner [2], have regarded Llull as giving rise to the modern theory of graphs but this seems hard to substantiate. Llull has been referred to as the "first computer scientist", in particular by Catalans but also in an extensive study by Künzel [19]. Ton Sales [36] describes Llull as "One of Us", where "Us" means Computer Scientists. Likewise Anthony Bonner [3] asks "What was Llull up to?" and presents a nice picture from the point of view of a modern computer scientist. But Llull himself would never, indeed could never, have described himself as a "computer scientist". The very phrase was not in circulation, and neither were the appropriate concepts current.

Llull developed his own way of doing logic, which is epitomized in his Great Art, *Ars magna*. In doing so he contributed to the logical notion of a formal language, and to the mathematical notions of variable and substitution (for a variable).[1] From today's point of view Llull made contributions to the following areas: 1. the idea of a formal language, 2. the idea of a logical rule, 3. the computation of combinations, 4. the use of binary and ternary relations, 5. the use of symbols for variables, 6. the idea of substitution for a variable, 7. the use of a machine for logic. He also pioneered the integration of ontology with logic.

Leibniz (1646–1716) mentions Llull by name several times and explicitly uses his ideas in [22]. Leibniz learnt of Llull's ideas through Kircher [18]. This work led directly into the development of computing machines. So Llull contributed ideas that are fundamental to the modern disciplines of computer science and computer engineering.

3.1 Llull's motivation

In looking at the contributions that Llull made and which led into computer science it is important to realize three things. First, that the construction of Llull's new logic in his Great Art took place throughout Llull's life.[2] Secondly, that Llull's principal aim in developing his system was not to develop logic but to provide a means for the conversion of monotheistic non-Christians, specifically Jews and Muslims (see [32]). Thirdly, the mechanical systems of Llull did not provide answers to the questions that his machine generated. Rather, having posed a question, Llull believed that it would then be possible to use logical argument to achieve the conversion of the non-Christian. As Colomer [8], p. 118, writes: "The logical mechanism of the Art is to serve a religious end." Thus Llull would begin in an objective world and

[1] When I refer to "Llull's methodology" I mean the way in which he obtained, or thought of, the devices he employed, rather than how to use Llull's system. For the latter there is an excellent guide in [4].

[2] In this article I shall only discuss the final version of Llull's magnum opus.

then ask questions. (See [26], p. XVII, and the *Prologue*, p. 3, of the *Ars Brevis*.) The objectivity would avoid the Christian taking a superior position and would promote engagement with the interlocutor. The solution of the questions would then convince the hearer of the truth of Christianity. However, I should stress that Llull's idea of logic is not purely abstract; it integrates facts with logic. (See also below Section 3.2.2.) As Rossi [34, p. 32], puts it: "While metaphysics considers entities external to the soul 'from the point of view of their being' and logic considers them according to the being which they have in the soul, the art —supreme among all the sciences— considers entities in both ways at once.[3]"

This aim was to be achieved by starting in an objective world, then asking questions and proceeding by pure logic to convince the non-Christians. Llull believed that starting from mutually acceptable premises, the logic of his arguments would convince the hearers of the truth of Christianity. In pursuit of this he spent a very large part of his life designing the appropriate (perfect, to use Eco's word in [11]) language.[4]

In order for his system to be effective it was necessary for it to be understood. The technicalities of his system meant that Llull was continually working to make it clearer and simpler. There was also the question of language: why should the interlocutor speak the Christian's language? This seems to be why he learnt Arabic.

It seems appropriate to regard Llull's work as being logical in the sense used by Leibniz, as well as in the contemporary sense (which is not too different). Llull himself says (*Ars Brevis*, see [26], chapter XII, Form 87):

> Logic is the art by which the logician finds the natural conjunction between subject and predicate.[5]

[3] Rossi has a note giving the reference in [24], vol. III, Int. ii, p. 1.

[4] Of course Eco was not the first or only person to use this adjective, see, e.g. [34].

[5] Besides predicates, which traditionally since Aristotle took only one argument (to use the modern terminology), Llull introduces binary and ternary relations. However, these are only in the context of his formulae, see Section 3.2.5 below.

3.2 Llull's formal treatment of language

In a way that presages the present day approach, Llull gave formal procedures for the language that he used. He presented a finite number of primitives.[6] Originally he started with many more but in the last analysis these were reduced to nine which form his *alphabet*: B, C, D, E, F, G, H, J, K.[7] These primitives will be interpreted in different ways. I discuss this in further detail below in Section 3.2.6 but for now it suffices to give an example: B is interpreted as goodness, difference, whether?, God, justice or avarice, C as size (or greatness), concordance, what?, angels, prudence, gluttony.

> B signifies goodness, difference, whether?, God, justice and avarice.
> C signifies magnitude, concordance, what?, angel, prudence and gluttony.[8]

Having determined the primitives he puts these together according to his four formats which he calls *figures* (see Section 3.2.1). One can justifiably say that he puts them together according to mechanical rules (see Sections 3.2.1 and 3.3 below). These combinations can then be read to give the various possible questions.

First I should distinguish between the *formulae*, namely the combinations of letters, on the one hand, and their interpretations. One is tempted to read his formulae as simple combinations BC, EK, etc. but this has to be interpreted in the context being used; Llull uses several distinct *figures*, see below, Section 3.2.1. Secondly, each individual letter is interpreted not only as an abstract noun, e.g. goodness, but also as the corresponding adjective, good. On occasion he goes further and makes verbs out of nouns. This is difficult to render in both Latin and English. Thus in chapter IX when dealing with the fourth subject, which is Man, we find:

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[6]In this and the following sections I shall principally restrict myself to Llull's *Ars Brevis* [26] since the points that I wish to make are all included there.

[7]The letter A had already been used for the name of Llull's first *figure*.

[8]It should be noted (for Section 3.2.3 below) that each letter has an interrogative word included in the list of interpretations, and also that one negative word is included.

> A man is composed of a soul and a body. For this reason he can be studied using the principles and rules in two ways: namely in a spiritual way and in a physical way. And he is defined thus: man is a man-making animal.

Following from this second point, I note that Llull explicitly interchanges subject and predicate. This is completely outside the normal practice of logicians throughout history.[9] This is already clear in the case of the first figure, which I discuss in the next section, and explicit in the treatment of the third figure in chapter VI, where he talks of "... exchanging subjects with predicates."

3.2.1 Llull's figures

Llull introduces *figures*, as he calls them, to yield combinations of letters. (See [13, pp. 301–310], or p. 10 above.) Figure A joins primitive elements by making one the subject and one the predicate.[10] Thus "goodness" is joined with "great", rather than with "size", to give "Goodness is great". Since there is no priority among the letters we can also join size—the abstract noun corresponding to "great"—with good[ness] to get "Size is good".[11] This also permits such conjunctions as "Avarice is good" which, since Llull says "Avarice is not good but evil", (*Ars generalis ultima*, [25, chapter I, 2]) shows that these are not true statements, but simply statements, or, as Llull generally uses them, questions. Thus Llull has provided a *syntax* for such statements.

The second figure, denoted by T, is more complicated to apply. (See [13, p. 309], or p. 11 above.) It comprises two segmented circles surrounding three triangles. The inner triangles each contribute in two ways to the formation of statements. First each side of a triangle connects two letters together; secondly it brings an

[9] Apart from Llull, this idea of treating subjects and predicates on the same level does not seem to have been used until the twentieth century when Henkin (1921–2006) introduced it in his paper [17].

[10] It is interesting that although Llull did not use suggestive notation, that is to say, dividing the alphabet into different parts for different uses, nevertheless he retained his letterings throughout his work. This perhaps explains why the figures themselves are labelled with letters quite remote from each other in the Roman alphabet.

[11] As mentioned above the noun may be replaced by the adjective.

additional quality into the formula. For example, in the first triangle we have *difference, concordance* and *contrariety*. Any one of these will record a type of conjunction between a letter of the alphabet and another one. For example, "goodness is different from greatness". I could symbolize this in modern guise by $\mathit{Diff}(x, y)$. Here Llull is introducing binary relations as opposed to predicates (see Section 3.2.5 below).

The third figure (see p. 12 above) continues this process, adding further complexity. This is where Llull's preoccupation with combinations emerges. The third figure exhibits all the combinations of two distinct letters from the Alphabet. Further Llull does not regard BC, for example, as different from CB and therefore the number of combinations is reduced to half. There are therefore ${}_9C_2$ combinations, i.e. the 36 that Llull has. Hidden under this is the fact that each letter of his Alphabet can also be interpreted according to the interpretation in either the first or second figures, as previously discussed.

In the fourth figure the process is continued. Here we genuinely have ternary relations (see also Section 3.2.5 below). He produces combinations of three letters, where there is a restriction on repeating a letter in the trio.

The details of the interpretation are not spelt out in his treatment of the fourth figure. This is reserved until he has introduced his definitions and rules: he gives the full and explicit interpretation when he describes his *Table*.

3.2.2 Principles: mixing logic and ontology

Llull's *principles* are what we would call *axioms*. For Llull these are not formulae but statements involving the terms of the alphabet. "For with such conditions the intellect acquires knowledge, … ." ([26, pp. 26–27]) So they are not purely abstract axioms. The first and tenth are:

> 1. Goodness is that whereby good does good.
> 10. Difference is that by reason of which goodness, etc. are distinct and clear notions.

The principles appear to have been chosen in the light of his experience perhaps in the same way that the laws of logic, even from the time of Aristotle, have been established.[12] That Llull, despite all his work, did not have the same idea as we have of the status of axioms is evident from his statement at the end of part 3 in the *Ars generalis ultima*, [25]. Here he claims that his principles help to clarify what is going on and to provide guidance in the way that a statement may be resolved to be true or false. To put it another way, Llull's ontology determines the acceptability of his principles.

> Now there are those who dare to attack our principles with canine fangs and serpentine tongue, as they disparage and slander our definitions. However, the Art has principles that mutually help each other, for instance when someone says: "If greatness is the being on account of which goodness is great, then all goodness must be equally great;" this can be refuted with the principles of majority, minority and contrariety which do not allow every kind of goodness to be equally great.

Today we would say that these rules are rather meta-rules, governing the working of the logic.

In his *Logica nova* Llull takes non-logical axioms, i.e. statements involving facts from the world as the basis for arguments (see [30]). In his later work he used the method of *demonstratio per hypothesim* where he takes apparently contradictory hypotheses and analyzes them further so as to remove the apparent contradiction (see [12]). Wyllie, see p. 123 below, has suggested that Llull is using a form of paraconsistent logic here.

[12] I am aware that the laws of logic are regarded as necessarily true, but modern logic has invented many different kinds of logics, with different laws, which are clearly valid in appropriate domains of discourse. Therefore I would maintain that although the laws of logic have withstood the test of time, nevertheless they are dependent on the kind of world in which we live, and the kind of creatures that we are. Llull thought over his principles for a very long time and gives no direct justification for them, while maintaining that they are essential. See also footnote 1, p. 309 of [1].

3.2.3 Rules

In a modern system of formal logic, having built the formulae we then have a set of rules by means of which we can deduce more formulae. The aim in modern logic is to go from true formulae to true formulae. Llull does indeed have rules, but their purpose is to generate more statements (or questions). However, they do not have the same form as rules of modern logic. Llull only allows joining basic letters (from his alphabet). The rules are purely syntactic; the semantic considerations are left entirely to the user. There are ten rules, one associated with each letter except there are two for K, and all using the question word associated with the letter concerned.

As noted earlier, every combination of letters gives rise to many different statements (or questions), for example, B and C using the first figure, and B, C and D from the fourth figure. Indeed, this whole process gives rise to a very large number of statements (or questions). Not only do we have the various possibilities for each letter, we also have different ways of ordering the letters. Llull provided a *Table* of the allowed combinations for his fourth figure (see p. 15 above).

3.2.4 Llull's *Table*

The *Table* lists all combinations of three letters from the fourth figure and each is put in a cell. In addition the letter T is included but it has a very different rôle from the letters B, . . . , K: it identifies the particular syntactical construction. Therefore, we get a sequence of four letters in each cell. T may occur in any of the four positions. Of the remaining letters none may be repeated on the same side of the T. Thus CTBD is permitted but CTBB is not.

All of this amounts to forming conjunctions, for example: CTBD which may be read: "C has B and D", can be interpreted as "Greatness has difference and contrariety". The general form of this kind of conjunction is "x has y and z".

Llull has therefore not only gone beyond *predicates* involving only one argument, such as Man(x): x is a man, to *relations* between two elements, such as x is less than y, which are called *binary* relations, but even to *ternary* relations, such as $x + y = z$

or "John, Mary and Peter constitute a family". So we can say that here Llull has provided a rule, which we could describe as generating formulae involving binary and ternary relations (see Section 3.2.5 below).

There is one special restriction. Expressions such as "C has B and B" are not permitted but "C has D and C" is. This latter is of the form: "x has y and x". Likewise those of the form: "x has x and y" are also permitted. Further, there is one major difference from modern usage. For Llull, two occurrences of x may be interpreted in different ways, i.e. chosen differently from the various possibilities, whereas in modern usage any substitution of something for x requires that the same object be substituted at all occurrences. In addition to this we also may find the interpretation in an adjectival form in one place and in a noun form in another. Thus "C has D and C" has an interpretation "Greatness has duration and gluttony". However it would be misleading to write x_1 has y and x_2 which suggests (to the modern reader) that x_1 and x_2 may be different: a syntactic difference. For Llull the underlying formula is the same, it is just that the same letter is interpreted in two different ways: a semantic difference.

When T is the first or last letter, the remaining three letters must be distinct. When there are three letters to be combined, there may be zero, one or two letters to the left of the T and the other letters at the other side. His use of the letter T indicates the bracketing. Thus BCTB could be written (BC)B in modern notation, whereas BTCB would be written B(CB). The combination TBCB is not permitted, but the combination TBCD puts all of B, C and D on a par. We could regard this as being B & C & D with our usual assumption that the associative law holds for conjunction. That is to say, it does not matter which & we use first. This bracketing makes the formulae read unambiguously. However, the interpretations are many since the letters on the left of the T are interpreted according to the first figure, and those to the right of the T according to the second. Llull presents a table of 1680 combinations for his Fourth Figure, because the combinations are not simply of three letters but are mediated by his restriction that the same letter should not occur twice on one side of the "T" and also because of the way that one can interpret the same letter in different ways.[13]

Having produced this large number of questions and statements, in his *Ars generalis ultima*, [25], Llull seems to think that he has covered all eventualities. He says of the table for the Fourth Figure: "This table is a subject in which the intellect achieves universality... ." To the modern reader this may be puzzling, since it is clear that the number of questions one might ask is without limit if one permits combinations of more letters. However, if we look at his way of proceeding, then we see that for each definite finite number,[14] in Llull's case, simply 2 and 3, he generates all the possibilities, that is to say, the pairs, triples, quadruples of letters.[15] The number of possibilities for each such definite number is finite because Llull has only a finite alphabet. This continues to be the case for any *specific* number of letters in a combination. It is only when one thinks of taking combinations involving arbitrarily large numbers of letters that there is the possibility of an infinite number of combinations —and there is no evidence that Llull ever had any reason to do that, nor that he even thought of doing so. He could already produce enough questions for his purpose of conversion to Christianity by using the combinations obtained from a small number of letters.

The idea of Llull of making more and more complicated combinations was taken up, as is well known, first by Kircher [18, 23] and then by Leibniz in his *Ars Combinatoria* [22]. Indeed Leibniz devotes considerable space to Llull. On [22, pp. 39–40] he is critical, as I was above, of Llull's restriction to nine letters. He asks in particular why Figure and Number are not included among Llull's basic terms. Then Leibniz proceeds to investigate combinations as such, developing his calculations to larger sets of basic elements and imposing conditions on repetitions. This was purely mathematical work. It was only later that Leibniz developed the idea of the *Ars Combinatoria* in the sense of being able to resolve logical disputes by taking up pens and calculating.

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[13] The best explanation of the number 1680 that I have found is in Eco [11], pp. 61–2.

[14] For a more precise discussion of the phrase "definite finite number" see my [9], Chapter I.

[15] Leibniz used the neologisms com3nation, com4nation, etc. for the combinations that involve only a specific number of letters in his [22].

3.2.5 Relations

It was Bonner who, in [3], first noted that Llull was using binary and ternary relations. He pointed his readers to the historical notes in [29], where, incidentally, Llull is not mentioned (see p. 14 above). Llull appears to be the first person to introduce binary and ternary *relations* as opposed to *predicates*. Before that time, and certainly in Aristotle, predicates were only unary. Although Llull does not directly allow arbitrary binary and tertiary relations, indeed he severely restricts them, nevertheless he does give schemata, which therefore means that there are several different relations. Thus in the third figure (see above Section 3.2.1), he has two types of binary relation. These differ only in that the order of the arguments is reversed. In Llull's notation this is the difference between the orders BC and CB. In the fourth figure there are ternary relations. These differ in the order of the letters and they are determined by the location of the letter T. Thus BTCB is very different from BCTB as I have noted above in Section 3.2.4. Even more different are BTCD and BCTD where we have three different letters not just two, but note our comment above in Section 3.2.4, about the different interpretations of two occurrences of a letter such as B in a Llullian formula.

Llull's move frees formal logic from the tyranny of relations (or should I say, predicates) having only one argument. Incidentally, once one has two or more arguments then it is possible to build more complicated relations, for example (in modern notation), $P(x,y)$ & $Q(y,z)$ builds a ternary relation between x, y and z, from two binary ones. Such constructions are impossible with predicates of one argument only. Llull does not go so far, so his sentences correspond to the forms $P(x,y)$ or $R(x,y,z)$ (with P and R being constant relations) and more complicated arrangements of relations are not considered.

3.2.6 Substitution

There has been significant discussion in the literature regarding the earliest date for the introduction of variables into mathematics.[16] Some authors have argued that variables were used by Diophantos early in the Christian era (perhaps *c*. 300AD). He was interested in the solution of numerical equations (as we would express it today). He used a symbol, which seems to be distinct from the letters of the alphabet, for an unknown. This may have been an abbreviation for a word meaning, in essence, "unknown". (See [9, p. 70]) However, the symbol did not have a variety of interpretations: it simply stood for a quantity that was required to be determined. Other symbols were used for various quantities. Diophantos used $\Box$ for the square of the unknown, i.e. the number multiplied by itself. This tradition continued through al-Khwârizmî [33] of the ninth century and into the Italian algebraists of the sixteenth century, where the unknown was called the *cos* or "thing". There was no suggestion that the value of an expression, say one equivalent to $x^2 + 10x$ might change, rather the emphasis was on solving problems expressible by equations such as $x^2 + 10x = 39$.[17] It is the sixteenth century Italian, Maurolico (1494–1575) [28], who is generally credited with the introduction of variables as mathematicians now use them. The full use of variables in algebra was not to emerge until the late sixteenth century in the work of Viète [40].[18]

In logic the use of letters as variables is hard to pin down, since the development was a very slow one.[19] We notice it most in work of Leibniz [21] in the seventeenth century.[20]

However we find something rather different from the modern view in Llull's use of letters. He uses the letters B, C, D, E,

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[16]The concept of variable is still difficult for school students to grasp, which may indicate that it is much more complicated than mathematicians usually assume.

[17]This, incidentally has the solution $x = 3$ but it also has a negative solution, which was not accepted for a long time, $x = -13$. See [33] and [9, p. 68].

[18]See especially his *De æquationum recognitione et emendatione*, [39] and [9, p. 99], [7, pp. 14–15]. In his *In artem analyticam isagoge* Viète did not exploit variables very much but he did use them to denote differing quantities as opposed to Cardano [6] (also sixteenth century), who only used letters to denote specific unknowns or powers (squares, cubes) of the unknown. For example on [39, p. 190], Viète is using variables as such.

[19]In mathematics it is usually attributed to the sixteenth century Italian mathematician Maurolico [28].

[20]Aristotle's use of letters seems to be of a different nature. In their cases each letter represents a specific, but perhaps unknown, element. In the case of Llull, Leibniz, etc. the same symbol (letter) takes on different values at different times.

F, G, H, J, K to stand for different things, but even in the same statement they may take on different values at different times.[21] This should be opposed to the situation in modern mathematics and logic where, for example, when we substitute for x in, say, $x^2 + 10x$, we replace each occurrence of x by the same number. For Llull, each letter has *several different interpretations*. There are therefore two differences even from the earlier uses in mathematics. First, the symbol was denoting *the* object, or number, to be found, and secondly, the symbol always represented this same number throughout the statement or argument.

Llull has not explicitly said so, but I would say that Llull was *substituting* words for these letters. Here I diverge from Bonner [3], see p. 8 above, who says "the letters don't represent variables but constants". Perhaps the best way to describe the situation is to say that there are several *different* constants corresponding to each letter. Moreover Llull has not just one possibility but a variety of possible substitutions: six in each case.

3.3 Machines

The idea of movement is very important in distinguishing between Llull's first figures and his fourth one. The involvement of movement in calculation is immensely ancient: for example, scratches on bone from Palaeolithic times (see [27]) involve movement. In historic times the system found in the work of the Venerable Bede (c. 673–735), allowed representations using various configurations of the hand for (individual) numbers up to 1 000 000.[22] Symbolism combined with, indeed directing, movement is found from the tenth century in the correlation by Guido d'Arrezo of points on the hand with musical intervals, and moving the finger of the other hand to point to these various locations. It is therefore surprising that Yates [43, p. 176] says: "Finally, and this is probably the most significant aspect of Lullism

[21] Variables were also used in literature to symbolize specific meanings. In Llull's time, the "Tower of Wisdom" of *c.* 1300 has twelve letters each of which has ten interpretations (see [37]).

[22] See, e.g., Science and Society Picture Library 2004 at `http://www.scienceandsociety.co.uk/results.asp?image=10308471&wwwflag=2&imagepos=1`

in the history of thought, Lull introduces movement into memory." What does seem to be the case is that Llull introduces a machine with moving parts into logic, and thereby, memory. It therefore seems both useful and indeed necessary to distinguish a "machine" as comprising a physical object in which there are parts that are movable with respect to each other, and this should be opposed to movement, in which the user moves separate individual objects, or his or her configuration relative to an object (as with the Guidonian hand).

Llull's first two figures had no moving parts. However the fourth figure had three discs independently revolving around a central axis. (See [13, p. 310] or [36] p. 34 above.) It therefore seems appropriate to call this a "machine". By rotating the discs, different alignments between letters (or words) on them yielded the differing combinations that I have referred to above.[23]

Thus the machines produced results.[24] These results then had to be interpreted (by substituting words for various letters) and these then gave statements (or questions). Such statements were sometimes true and sometimes false. However the machines did not determine which were which. It can therefore be said that the machines aided the logic (and the logician) but they did not do all the work; they did not perform all the calculations. What he did produce was a system and a machine for generating sentences: that is to say, statements or questions.

I would therefore argue that it is accurate, and in accordance with modern usage, to say that Llull "computed results". But I would also stress that the results that he computed were not the final word. In addition to his computation process he had to provide (or wished the users of his system to provide) the necessary arguments to determine the truth or otherwise of the statements

[23] Llull used eight discs when he discussed conjunctions of planets and the zodiac, see [35, p. 349–350].

[24] It is perhaps worth mentioning that the question of input and output from any machine is a separate question from the workings of the machine. This is as true for today's computers as it is for Llull's machines. For a modern machine there has to be an input provided. This is usually done, or at least, stimulated, by punching in various characters on a keyboard. The keyboard is not part of the computation. Likewise, when we need an output this is achieved, normally, either by getting an image on a screen or by printing something out on paper. Again these are not part of the computation process.

generated from his computations.

On pp. 51–52 of Gerhardt's edition of the *Ars Combinatoria*, [22], Leibniz discusses the "wheels" (*rotae*) of Llull and his successors. Later Leibniz was himself to design a machine for computing, in this case, for the four basic arithmetic operations $+, -, \times, \div$ using wheels. There seems to be a much stronger case for calling this a "computer" than there is for so labelling Llull's machine.

I also note that Leibniz included a *Proof of the existence of God* in his work *Ars Combinatoria*. This seems to have been added as a separate exercise. It is listed at the very end of Leibniz's list of contents yet occurs at the beginning of the work. This "proof" does not use combinations at all, but it does use the kind of logical argument that Llull uses to search out the truth of statements produced by his machines. Such an argument goes from axioms in a logical way, quoting which axioms, or previously deduced statements, are being used at each stage of the proof. This clearly foreshadows the rules of formal logic and the style of reasoning that we use today, which were introduced by Boole, Frege and Russell [5, 14, 41]. Leibniz writes on formal logic with a symbolism that is close to today's in his manuscripts [21] and [20].[25] These works are self-contained and do not in fact refer to Llull, but it is hard to imagine that Leibniz's beginnings of logic in terms of a formal language were not also influenced by Llull.

3.4 Conclusion

Claims have been made that Llull was the first computer scientist. Martin Gardner [15] wisely only refers to Llull making "logic machines", not "computers". Llull certainly made machines. He was also involved with the design of what is now described as a "formal language". The machines did actually compute results. The fact that they were relatively simple results: the combinations of two or three letters (from an alphabet of, in the last analysis,

[25]Leibniz's work in logic remained unknown for centuries. In the nineteenth century we find Boole [5], developing what was to become the propositional calculus. I have found no other acknowledgment of this work of Leibniz before 1900. Boole worked in the nineteenth century, and although Boole had access to some of Leibniz's work, it seems that it did not have any influence upon him. See Grattan-Guinness [16, p. XLIII], and also the *Encyclopedia Britannica* article by Dipert [10].

nine), should not detract from Llull's inventiveness. I therefore conclude that Llull could be described as a "computer engineer" in that he made logic machines which, after all, was what Turing [42] did last century.

I agree with Colomer [8], p. 132, who writes:

> First there is a coincidence of external appearance, but with incalculable consequences for the future: the formalization of language, that is to say, the creation of an artificial language, in which signs replace operations of ordinary language.[26]

So Llull made significant contributions to what is now known as "computer science" in that he developed the idea of formal language (formal combinations of symbols), contributed to the use of variables and introduced, albeit to a limited extent, binary and ternary relations.

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[26] "Hi ha primerament una coincidència de caràcter extern, però de conseqüències incalculables cara al futur: la formalització del llenguatge, o sigui, la creació d'un llenguatge artificial, en el qual els signes substituteixen les operacions del llenguatge comú."

Bibliography

[1] Anthony Bonner. *Doctor Illuminatus: A Ramon Llull reader*. Mythos. Princeton University Press, Princeton, NJ, 1993. Translated and edited by A. Bonner, with a new translation of *The Book of the Lover and the Beloved* by Eve Bonner.

[2] Anthony Bonner. Ramon Llull: Relació, acció, combinatòria i lògica moderna. *Studia Lulliana*, 34:51–74, 1994.

[3] Anthony Bonner. What was Llull up to? This volume, pp. 5–24.

[4] Anthony Bonner. *The Art and Logic of Ramon Llull: A User's Guide*, volume 95 of *Studien und Texte zur Geistesgeschichte des Mittelalters*. Brill, Leiden, 2008.

[5] George Boole. *An investigation of the laws of thought on which are founded the mathematical theories of logic and probabilities*. Dover, New York, 1958. Originally published in 1854.

[6] Girolamo Cardano. *The great art; or, The rules of algebra: Artis Magnae sive de Regulis Algebraicis*. M.I.T. Press, Cambridge, MA, 1968. Translated and edited by T. Richard Witmer, with a foreword by Oystein Ore.

[7] Giovanna Cleonice Cifoletti. *Mathematics and rhetoric: Peletier and Gosselin and the making of the French algebraic tradition*. PhD thesis, Princeton University, Princeton, NJ, USA, 1992.

[8] Eusebi Colomer i Pous, SJ. De Ramon Llull a la moderna informàtica. *Estudios Lulianos*, 23 (68–69):113–135, 1979.

[9] John Newsome Crossley. *The Emergence of Number*. World Scientific, Singapore, 2nd edition, 1987. First edition, UDA Book Company, Steel's Creek, Australia, 1980.

[10] Randall R. Dipert. Logic, history of [electronic version]. In *Encyclopædia Britannica Online*, 2006. `http://search.eb.com/eb/article-65939`, accessed 1.vii.06.

[11] Umberto Eco. *The search for the perfect language*. Blackwell, Oxford, 1995. Translated by James Fentress.

[12] Alexander Fidora. Concepts of Philosophical Rationality in Inter-Religious Dialogue: Crispin, Abelard, Aquinas, Llull. In Alessandro Musco, editor, *Proceedings of the XII International Congress of Medieval Philosophy, organized by SIEPM - Société Internationale pour l'Étude de la Philosophie Médiévale, Palermo, 17–22 September 2007*. To appear in 2011.

[13] Alexander Fidora and Josep Enric Rubio, editors. *Raimundus Lullus: An Introduction to His Life, Works and Thought*. *Corpus Christianorum Continuatio Mediaeualis*, 214, Supplementum Lullianum II. Brepols, Turnhout, 2008.

[14] Gottlob Frege. *Begriffsschrift und andere Aufsätze*. Olms, Hildesheim, 2nd edition, 1995. Notes by E. Husserl and H. Scholz, edited by I. Angelelli.

[15] Martin Gardner. *Logic machines and diagrams*. McGraw-Hill, New York, 1958. 2nd edition, Harvester Press, Brighton, 1983.

[16] Ivor Grattan-Guinness. *The search for mathematical roots, 1870–1940: logics, set theories, and the foundations of mathematics from Cantor through Russell to Gödel*. Princeton University Press, Princeton, NJ, 2000.

[17] Leon Henkin. Completeness in the Theory of Types. *Journal of Symbolic Logic*, 15:81–91, 1950.

[18] Athanasius Kircher. *Ars magna sciendi*. J. Jansson, Amsterdam, 1669.

[19] Werner Künzel and Heiko Cornelius. *Die Ars Generalis Ultima des Raymundus Lullus, Studien zu einem geheimen Ursprung der Computertheorie*. Advanced studies in modern philosophy and computer science. Berlin, 5th revised and expanded edition, 1991.

[20] Gottfried Wilhelm Leibniz. PHIL., VII, B, II 70–71: Essais de Calcul logique. In Louis Couturat, editor, *Opuscules et fragments inédits de Leibniz, Extraits des manuscrits de la Bibliothèque royale de Hanovre*, pages 264–269. Olms, Hildesheim, 1961. Reprint of the 1890 edition.

[21] Gottfried Wilhelm Leibniz. XIX. [Non inelegans specimen demonstrandi in abstractis]. In C. I. Gerhardt, editor, *Die philosophischen Schriften von Gottfried Wilhelm Leibniz*, volume VII, pages 228–235. Olms, Hildesheim, 1961. Reprint of the 1890 Berlin edition.

[22] Gottfried Wilhelm Leibniz. Dissertatio de Arte Combinatoria. In C. I. Gerhardt, editor, *G. W. Leibniz: Mathematische Schriften*, volume V, pages 7–79. Olms, Hildesheim, New York, 1971.

[23] Thomas Leinkauf. *Mundus combinatus. Studien zur Struktur der barocken Universalwissenschaft am Beispiel Athanasius Kircher SJ (1602–1680)*. Akademie Verlag, Berlin, 1993.

[24] Ramon Llull. *Raymundi Llulli Opera omnia*. Mainz, 1721–1742. Ed Ivo Salzinger *et al*. 8 vols. Reprinted Frankfurt am Main 1965.

[25] Raimundus Lullus. *Raimundi Lulli Opera Latina, XIV, [Opus] 128 Ars generalis ultima*. Corpus Christianorum Continuatio Mediaevalis LXXV. Brepols, Turnhout, 1986. Lugduni MCCCV incepta Pisis Anno MCCCVIII ad finem perducta. Edidit Aloisius Madre. Available at `http://lullianarts.net/ArsGeneralisUltima.pdf`, accessed 5.iii.2009.

[26] Raimundus Lullus. *Ars brevis, Lateinisch–deutsch*. Felix Meiner, Hamburg, 1999. Übersetzt, mit einer Einführung herausgegeben von Alexander Fidora.

[27] Alexander Marshack. *The roots of civilization: the cognitive beginnings of man's first art, symbol and notation*. Moyer Bell, Mount Kisco, NY, 1991. Rev. and expanded from 1972 edn.

[28] Francesco Maurolico. Arithmeticorum libri duo. In *Opuscula Mathematica*. Francisci, Venice, 1575.

[29] Anil Nerode and Richard Shore. *Logic for Applications*. Graduate texts in computer science. Springer, New York, 2nd edition, 1997. References are to the first edition of 1993.

[30] A. Oliver and M. Senellart (eds.). Liber de nouo modo demonstrandi. In *Raimundi Lulli Opera Latina XVI*, volume 78 of *Corpus Christianorum Continuatio Mediaeualis*, pages 339–377. Brepols, Turnhout, 1988.

[31] Carl Prantl. *Geschichte der Logik im Abendlande*. Akademische Druck- und Verlagsanstalt, Graz, 1955. 4 vols. in 3: vols. 1, 3 and 4 reprint of the 1855-70 ed., vol. 2 reprint of the 2nd ed., 1885.

[32] Robert Duguid Forrest Pring-Mill. The analogical structure of the Lullian art. In Samuel Miklos Stern, Albert Habib Hourani, and Vivian Brown, editors, *Islamic philosophy and the classical tradition: essays presented by his friends and pupils to Richard Walzer on his seventieth birthday*, pages 315–326. University of South Carolina Press, Columbia, SC, 1973.

[33] Frederick Rosen. *Muhammad ibn Musa al-Khwarizmi: Algebra*. Oriental Translation Fund, London, 1831. Reprinted by Olms, 1986.

[34] Paolo Rossi. *Logic and the Art of Memory: The Quest for a Universal Language*. Continuum, London, New York, 2006, originally published 2000. Tr. with intro. by Stephen Clucas.

[35] Josep Enric Rubio. The Natural Realm. In [13], pp. 311–362.

[36] Ton Sales. Llull as computer scientist or why Llull was one of us. This volume, pp. 25–37.

[37] Lucy Freeman Sandler. John of Metz, *The Tower of Wisdom*. In Mary Carruthers and Jan Ziolkowski, editors, *The Medieval Craft of Memory*, pages 215–225. University of Pennsylvania Press, Philadelphia, 2002.

[38] John F. Sowa. *Knowledge Representation: Logical, Philosophical and Computational Foundations*. Brooks/Cole, Pacific Grove, CA, 2000.

[39] François Viète. De æquationum recognitione et emendatione. In Franciscus à Schooten, editor, *Opera mathematica*, pages 82–158. Elzevier, Leiden, 1646. Foreword and index by Joseph E. Hofmann, Reprinted Olms, Hildesheim, 1970.

[40] François Viète. *The Analytic Art*. Kent State University Press, Kent, OH, 1983. Trans. by T. Richard Witmer.

[41] Alfred North Whitehead and Bertrand Arthur William Russell. *Principia Mathematica*. Cambridge University Press, Cambridge, 1925–27. 3 vols. First published in 1910–13.

[42] David Yates. *Turing's Legacy: A History of Computing at the National Physical Laboratory, 1945–1995*. Michigan State University Press, East Lansing, MI, 1997.

[43] Frances A. Yates. *The Art of Memory*. Routledge and Kegan Paul, London, 1966.

4 From *De arte electionis* to Social Choice Theory

Josep M. Colomer

4.1 Introduction

Ramon Llull should be considered the founding father of voting theory and social choice theory. He wrote (at least) three seminal pieces on voting and elections, to be cited using the abbreviations given in the parentheses:

- *Artificium electionis personarum* (c. 1274) (The method for the elections of persons) (AEP)

- *En qual manera Natana fo eleta a abadessa* (1283) (In which way Nathana was elected abbess) (B24)

- *De arte electionis* (On the method of elections) (1299) (DAE)

At least the second and third pieces were originally written in Catalan. In AEP Llull presents his initial voting proposal in scholarly form while introducing a graphic representation of binary comparisons that predates modern uses in computer science. B24 is the 24$^{\text{th}}$ chapter of *Blaquerna*, considered one of the first novels written in a Romance language, where the election of the abbess of a convent is used as an occasion for divulgation. A variant of the system with significant differences in likely performance is scholarly presented in DAE. (The three texts with translations into English are available entirely in [25, 17] and partly in [32, 33, 34, 12]).

4.2 The emergence of majority rule

The immediate context of these Lullian writings was a period of frequent and dramatic conflicts in the Christian Church on the occasion of elections of bishops, abbots, abbesses, priors and other prelates including popes. These elections were traditionally guided by the principle of unanimity, as voting was conceived as a way to reveal God's will and discover the truth. In the 5th century the Church had formally adopted from the Justinian code of Rome, which established that "What concerns similarly all ought to be approved by all", the principle that "He who governs all should be elected by all". But very often a unanimity agreement was not reached by the faithful, the clergy or the friars with voting entitlements, thus provoking recurrent conflicts and schisms. As the arbitral power of the Roman-German emperor and other traditional local rulers began to dwindle, the Church was moved to choose more effective voting rules on its own.

More specifically, a less-than-unanimity rule, namely 2/3, was generally established for Church elections in 1179. Three specific procedures were devised in 1215: "acclamation" (by at least 2/3 of the voters participating in any voting for a candidate), "compromise" (involving the transfer of the election to a previously elected committee, which should also decide by 2/3), and "scrutiny" (where in the case of no clear decision, the "sanior [actually senior or sounder] part" of the electorate should prevail). Furthermore, for the election of the pope by 2/3 a conclave of cardinals was designed in 1276. At its establishment, pope Gregory X finally acknowledged that "Not zeal to zeal, nor merit to merit, but solely numbers to numbers are to be compared" (in [24, VI 9]; more details in [11]; also [43, 44, 5, 12, 48]).

Echoing these practices, Llull made explicit references in his writings to the aim of preventing "fraud and simony" (AEP), the advantages of his "new electoral method" in contrast to "the method to which they were accustomed for an election" (B24), and his intention to be "far from the secret scrutiny and the election by compromise wherein more fraud can be committed than in [his] proposed method" (DAE). "Evidently —he stated— a good electoral system in the Holy Church is very necessary for electing office holders, since by them the church is governed and they fight against the church's enemies who commit sins like the infidels and schismatics" (DAE).

More generally, highly innovative elections by majority rule were gradually accepted in late medieval times as an expedient system when unanimous and consensual decisions resulted impossible. Some forms of majority decisions had been used in ancient Roman comitia and councils, but actually they involved mixed procedures of indirect elections, lots and shows of allegiance by acclamation. Traditional unanimous or broadly consensual decisions in old local communities in disparate places had been achieved by such procedures as silent acquiescence, clashing spears against shields, shouts of commendation or acclamation at harangues, murmurs in favor or cries against the proposer, rising on one's feet, or other 'viva voce' expressions, rather than formal voting sessions [6, pp. 13–30].

By the time of Llull's writings, voting and elections were regularly held in Christian convents and councils, as well as in many city councils and representative parliaments all across Europe. Different voting rules and procedures involving people's acclamation, lots, several stage elections, approval voting, and eliminatory methods were innovatively tried [28, 4, 37, 8]. In particular, Llull must have been aware that in contrast to most other institutions of the kind, in the Parliament of Catalonia majority rule prevailed among clergy and cities (while the noble estate stuck to unanimous requirements), [16, 30, 19] and [5, pp. 77–79].

The establishment of the majority principle in late medieval and early modern times, especially in medieval Germanic law and the Church's canon law, turned out to be one of the greatest achievements in human civilization. It implies nothing less than the provision that every member of a community ought to consent to collective decisions that he or she may not share. Majority rule is much more effective in making collective decisions than the traditional unanimity rule by which every individual can veto any decision. At the same time, if it is soundly enforced it can prevent despotism and oligarchy. Llull's proposals are the first serious attempts to devise systems able to implement the majority principle in real elections. This is why his works can be considered of foundational and precursory nature [26, 31, 6].

Llull's voting proposals were, however, forgotten for several centuries. With minor variants, they were somehow reinvented in the process of establishing innovative democratic elections in late 18^{th}-century revolutionary France. While Llull's B24 was available in Catalan and other languages for a long term (see for instance, [20]), DAE was discovered in 1937 [27] and AEP in 1959 [39], but nobody with interest in voting and elections paid attention to these works for a while. Actually Llull's basic voting procedure was reinvented by the mid-20^{th} century. And his works were eventually rediscovered in the context of worldwide diffusion of electoral democracy and the development of social choice theory. (Especially by [32, 33, 34, 25]. Further references in [18, 5, 6, 7]. See, however, how encompassing analyses of some of those procedures refer only to Copeland, but not to Llull, as, for example, in [46, 47]).

4.3 Exhaustive Binary Comparisons

Ramon Llull's initial proposal (as presented in AEP and B24) is a voting system based on decisions by majority rule in exhaustive pair-wise or binary comparisons of all candidates. This approach is derived from Llull's general method or "Ars generalis" by which exhaustive combinations of a small number of simple basic principles or categories would be able to account for all wisdom in every branch of human knowledge. In his writings on voting, Llull, nicknamed Doctor Illuminatus, uses tables with graphic representations of all exhaustive combinations in a similar way to many others of his works in which diagrams, connecting lines and rotating circles abound. For this (and his influence on Gottfried Leibniz) he has also been considered a precursor of computer science [21, 3].

According to Llull's voting system, an election is to be made by holding multiple rounds of voting by majority rule between all possible pairs of candidates. For n candidates, this requires $n(n-1)/2$ comparisons. The winner is the candidate winning by majority in the greatest number of binary comparisons. In Llull's words:

> In each voting round or comparison between two candidates, "a point is given . . . to the person who has the

> greatest number of votes"; after all rounds, "the person with more points is elected" (AEP); "let be elected the one who will have the greatest number of votes in the greatest number of chambers [cells or comparisons]" (B24).

These sentences should help clarify some previous confusion with other procedures such as rank-order count. In the present author's translations from Latin and Catalan, respectively, both "plures voces" in the first text and "més veus" in the second are translated as "the greatest number of votes", not as "the most votes". Llull proposes to make a winner the candidate winning the greatest number of votes in the greatest number of comparisons, not in all comparisons added together. Llull's basic system was reinvented by the American mathematician Arthur H. Copeland by the mid-20th century [15].[1] In contrast, the rank-order count system requires the voters to rank all the candidates and award them 0, 1, 2, etc. ordinal points from the least to the most preferred; the winner is the candidate collecting the highest sum of points. This system was not proposed by Llull but firstly devised by Nicholas of Cusa (1401-1464), precisely after he transcribed Llull's *De arte electionis*. The rank-order count system was also reinvented by Jean-Charles de Borda (1733-1799) in the French revolutionary context.

A very similar system to Llull's original system is the better-known Condorcet system, as proposed by Marie Jean Antoine Nicolas de Caritat, marquis de Condorcet (1743-1794), who was chairman of the revolutionary French National Assembly and author of the Girondin party's constitutional project in the Convention (Condorcet 1785). As is well known, by the Condorcet system a candidate is required to win by majority all binary or pair-wise comparisons, that is, the Condorcet-winner is the candidate able to win by majority against each and every other candidate —a rather compelling "majority" winner indeed.

[1] A minor difference exists. In every binary comparison, Llull proposed giving one point to the winner by majority, as well as one point to each of the two candidates if a tie of votes arises, and add up the points for each candidate. Copeland proposed giving one point to the winner and half a point to each of the two candidates in a tie. Llull, nevertheless, advises gathering an odd number of voters in order to prevent ties.

It has been noted that when the voters vote on the basis of their relative "distances" to the candidates and the latter are perceived as ordered along a single linear dimension (such as the left-right axis that emerged for the first time as a representational device in the French National Assembly), the Condorcet-winner is always the candidate preferred by the median voter. As the median voter's preference minimizes the sum of distances from all voters' preferences, the Condorcet-winner can be considered highly satisfactory for the electorate and is frequently used in social choice theory and voting analyses as a positive reference for comparison with the winner by other voting systems. (See extensive discussion in [5]).

However, in multidimensional issue spaces a candidate capable of winning against every other candidate may not exist. A Condorcet "cycle" can occur by which every candidate can be defeated by some other, such as in X>Y>Z>X . . . (where each letter represents a candidate and ">" means "defeats"). Thus the Condorcet system can be inefficacious in producing a single winner.

Obviously, the Llull system produces the same winner as the Condorcet system when the latter exists. Generally, the Llull system should be more efficacious in producing a winner than the Condorcet system, as it does not require winning all but only a higher number of binary comparisons than the other candidates.

Let us see an example of how these systems work in Table 4.1, where I try to replicate the Lullian kind of binary combinations, cells and matrices that can evoke modern methods in computer science. There are five candidates, V, W, X, Y, Z, and five voters, A, B, C, D, E, with complete orders of preference over the candidates. By casting all $5(5-1)/2 = 10$ binary comparisons of all candidates and assuming that the voters vote for their most preferred candidate within each pair, candidate V wins in 3 comparisons (against W, X, Y), W wins in 2 (against X, Y), X wins in 2 (against Y, Z), Y wins in 1 (against Z), and Z wins in 2 comparisons (against V, W). So, no candidate wins against every other candidate, thus there is no Condorcet-winner, but a cycle by which V>W>X>Y>Z>V. . . The Condorcet system does not produce a winner. The Llull system, in contrast, does produce a winner, V, who wins in 3 comparisons, with no ties.

Voters:	A	B	C	D	E
First preference	V	Z	Z	X	V
	W	V	V	W	X
	Y	W	W	Y	Y
	X	X	X	Z	Z
Least preference	Z	Y	Y	V	W

Binary comparisons	*No. of comparisons won*
V(4) > W(1); V(4) > X(1); V(4) > Y(1); V(2) < Z(3);	V: 3
W(3) > X(2); W(4) > Y(1); W(2) < Z(3);	W: 2
X(4) > Y(1); X(3) > Z(2);	X: 2
Y(3) > Z(2)	Y: 1
	Z: 2
	Total =10

The numbers in parenthesis indicate votes, according to the voters' preferences; ">" means "defeats".
Results: No Condorcet-winner, but Condorcet cycle: V>W>X>Y>Z>V . . .
Llull-Copeland winner: V.

Table 4.1: Condorcet and Llull Systems

However, the Llull system is more effective in producing a winner than the Condorcet system only in elections with five or more candidates, such as in the example just presented. Let us assume, in contrast, that the election is contested by $n = 4$ candidates: W, X, Y, Z. There are, thus, $4(4-1)/2 = 6$ pairs of comparisons: W-X, W-Y, W-Z, X-Y, X-Z, Y-Z. In order to win by the Condorcet system, a candidate should win in 3 pairs (that is, against each and every one of the other 3 candidates). If there is no Condorcet-winner, it must be because no candidate wins in more than 2 pairs. But then, with 4 candidates and 6 comparisons, at least two candidates must tie by winning in 2 pairs each (for example, W may win in 2 pairs, X in 2 pairs, Y in 1 pair, and Z in 1 pair, or three candidates may win in 2 pairs each).

Then, for 4 or fewer candidates, if no Condorcet-winner exists, the Llull system does not produce a winner either, but a tie in the number of comparisons won by more than one candidate, thus not solving the inefficacy of the Condorcet system. Actually, with five or more candidates, the Llull system, as it does not require winning all comparisons, can also produce a tie in the number of comparisons won by more than one candidate. The Llull system can, therefore, produce relatively frequent ties in relatively complex electorates. "Complexity" may imply either a multidimensional issue space prone to producing a Condorcet cycle (even with a low number of candidates) or a high number of candidates. Otherwise, with moderate numbers of issue dimensions and candidates the Llull system tends to match the Condorcet results.

In his writings Llull presented efficacious examples of elections with more than four candidates (16, 7 [as previously selected by a broader electorate] and 9 candidates, respectively in AEP, B24 and DAE). In AEP, nevertheless, Llull provided a subsidiary system to break ties: "lots are drawn . . . and the one [candidate] whose lot wins is elected".

The election of officers by lots had a long tradition in classical and medieval cities, as well as in the early days of the Church. The first apostles of Jesus drew lots to select the replacement for the traitor Judas (Acts of the Apostles I: 23-26). They were imitated by early non-orthodox Gnostic Christians, who drew lots at each of their meetings to elect priests, bishops and other officers. But the Christian Church condemned such a practice as blasphemy and solemnly forbade the choice of prelates by lots. In the 13^{th} century, just a few years before Llull's writings on voting, lots were formally prohibited in a new canon law collection formed by Raymond of Penyafort which was given force by pope Gregory IX's bull *Rex Pacificus* (1234): "Not only should election by lots not be done, but the delegates in elections by compromise should not be selected by lots" (in [23, XXI, III, 823]; related discussion in [6, pp. 22–25] and [9]).

4.4 Non-exhaustive Comparisons

In the days before printing and other modern communication techniques were available Ramon Llull may have been inadvertent for a little while of the formal papal prohibition of lots for elections of prelates in the Church. Actually he contemplated lots only to break ties between candidates, not to elect them in first instance. Nonetheless, in further works he withdrew this proposal. In B24 Llull resolved that in the event of a tie "one decides solely by means of the system", that is, without making resource to another system, such as lots, and chooses the candidate with best love of God, best virtues, least vices, and being the most suitable person (although he left open the question of who should judge these qualities).

More innovatively, in DAE Llull presented a new voting system that is able to prevent ties (and thus avoid the need to resort to lots or other problematic formulas). This is a simplified, non-exhaustive binary comparisons system by which after every round of voting the loser candidate is eliminated, the winner by majority is then compared with another candidate, and so on, and the elected is the candidate winning the last comparison. This system needs only $n - 1$ binary comparisons. For instance, in the above illustration with four candidates, there would be only 3 comparisons instead of 6, say, for example, W-X, X-Y, Y-Z (in the assumption that W<X, X<Y, and then the elected will be the winner in the comparison Y-Z).

Llull apparently expected that this system would produce the same winner as the previously presented exhaustive series of comparisons and, in addition, would prevent ties and be more efficacious. Obviously, if a Condorcet-winner exists, both the initial Llull system and the simplified-eliminatory Llull system produce the same winner. If the Condorcet-winner does not exist, for four or fewer candidates the simplified-eliminatory Llull system can indeed prevent ties and be more efficacious than the initial Llull system. However, for any number of candidates the simplified-eliminatory method can produce different winners depending on the order in which the candidates are compared.

The previous example in Table 4.1 with five candidates in which there is no Condorcet-winner can illustrate this point. While the initial Llull system makes V the winner, the simplified-eliminatory Llull system can produce either the same or a different winner. Specifically, if V is paired with Z, the latter will win and V will be eliminated. The final winner can be any of the five candidates depending on the order in which the candidates are compared (since every one of them is capable of winning at least against one other candidate).

The eliminatory voting system, also called "successive procedure", is actually used nowadays in most state parliaments in Europe for regular voting on motions, bills and amendments. By this procedure, each of the amendments is voted 'yes' or 'no' and consequently adopted and added to the bill or eliminated. This system has been critically analyzed by social choice theorists at least since the mid-20th century. It has been established that the order in which the alternatives are presented can change the winner as long as there are fewer voting rounds than alternatives. This is certainly the case of the simplified-eliminatory Llull system, which envisages $n-1$ rounds of voting for n candidates. It is also the case of the parliamentary successive procedure just mentioned which involves n alternatives, counting the initial bill and the bill with each of the proposed amendments, and only $n-1$ voting rounds for accepting or rejecting the amendments and approve the bill, whether amended or not, by default [40]. Regarding the potential winner, "the later any motion enters the voting order, the greater its chances of adoption" [2, p. 40]. The organizers of the voting session can, thus, manipulate the system in order to achieve some preferred outcome.

4.5 Secret or Open Ballot

Ramon Llull also discussed the interesting question of whether voters should vote in secrecy or openly in order to favor his aims of achieving the best electoral result. According to conventional uses in the Church of his time, he admonished that "each person having a vote in the chapter should take an oath by the holy gospels of God" to consider the moral, intellectual and personal characteristics of the candidates "to always elect the person in whom they are best [embodied]" (AEP). In B24 Llull specified that "an oath be taken by all the [voting] sisters to tell the truth", and then they would vote "in secrecy" for the best candidate. This provision implied some fear of coercion by more powerful voters, whether in hierarchy or perhaps as members of the "sanior" part of voters, capable of distorting the voters' sincere revelation of preferences. With these concerns, Ramon Llull anticipated arguments in favor of the secret vote by about six centuries.

However, worried about fraud in traditional secret elections in the Church, he eventually leaned towards the side of the open vote. He argued that while "those who elect publicly face great disgrace by their colleagues if they elect badly; those who elect secretly do not" (DAE). With this twist, Llull also predated further relevant discussions, particularly on the role of the "tribunal of the public opinion" that could be capable of overcoming coercive pressures from the powerful few (as it was developed, among others, by Jeremy Bentham and John Stuart Mill in 19^{th}-century England, see [29]).

4.6 Comparing Voting Systems

Different systems can be evaluated for their likelihood of producing the same or different results. A preliminary comparison between the Llull-Copeland system based on exhaustive binary comparisons and the Cusanus-Borda system is shown in Table 4.2. The total numbers of points for each candidate in the Cusanus-Borda count are the same as the numbers of votes in the exhaustive binary comparisons —which in this example are able to produce a clear winner with the Llull system. This result also holds when a Condorcet-winner exists. Thus, the Cusanus-Borda ranking of candidates equals the Llull ranking. This may suggest relative closeness between the systems under scrutiny. However, different winners can be produced by them for certain distributions of voters' preferences, as will be illustrated below.

More generally, a number of desirable but sometimes difficult-to-achieve properties for voting systems have been established and discussed at length, such as those summarized in Table 4.4. They include, first, certain properties regarding the outcome, such as the capacity of always electing the majority winner (that is, the candidate who is the first preference of an absolute majority of voters) or the Condorcet-winner if it exists, and of never electing the majority loser (the candidate who is the least preference of an absolute majority of voters) or the Condorcet-loser if it exists. Other properties refer to whether the outcome ought or ought not to be affected by changes in the number of candidates (the winner ought to be "independent on irrelevant alternatives") or in the amount of voters' preferences for a candidate ("monotonicity" or positive response to changes in voters' preferences).

The voting systems reviewed in Table 4.4 are: plurality rule (requiring only a relative, not an absolute majority of votes), majority runoff (including a second round between the two most voted candidates if they are short of a majority), approval voting (by which voters can vote for any number of candidates), the Llull-Copeland, Condorcet and eliminatory procedures presented above (based on binary comparisons of candidates), the rank-order count discussed above, and Bentham's range voting (permitting different numbers of votes to candidates).

Social choice theorists tend to remark that no voting system is perfect. Indeed, as can be seen in the Table, no voting system fulfills all desirable properties. This means that none of these voting systems can guarantee that all these properties will always be fulfilled with whatever number of candidates and distribution of voters' preferences. This kind of finding led some founding authors of modern social choice theory to formulate several "impossibility" theorems [1, 22, 45]. (For other comparisons of voting systems see [42, 38, 35]).

However, it has also been discussed that not all normative

Voters:	A	B	C	D	E	Rank Points
First preference	V	Z	Z	X	V	4
	W	V	V	W	X	3
	Y	W	W	Y	Y	2
	X	X	X	Z	Z	1
Least preference	Z	Y	Y	V	W	0

Binary comparisons	*No. of comparisons won*
V(4) > W(1); V(4) > X(1); V(4) > Y(1); V(2) < Z(3);	V: 3
W(3) > X(2); W(4) > Y(1); W(2) < Z(3);	W: 2
X(4) > Y(1); X(3) > Z(2);	X: 2
Y(3) > Z(2)	Y: 1
	Z: 2
	Total =10

Votes in binary comparisons:	Points in rank-order count:
V (4+4+4+2) = 14	V = (4+3+3+0+4) = 14
W (1+3+4+2) = 10	W = (3+2+2+3+0) = 10
X (1+2+4+3) = 10	X = (1+1+1+4+3) = 10
Y (1+1+1+3) = 6	Y = (2+0+0+2+2) = 6
Z (3+3+2+2) = 10	Z = (0+4+4+1+1) = 10

Comparisons won, or Llull-Copeland rank:	V (3) > W (2), X (2), Z (2) > Y (1)
Votes in binary comparisons:	V (14) > W (10), X (10), Z (10) > Y (6)
Rank-order count, or Cusanus-Borda rank:	V (14) > W (10), X (10), Z (10) > Y (6)

Table 4.2: Equivalence of Voting Systems

properties enunciated by theorists may be equally desirable. This normative discussion is relevant because, as can be seen in the Table, different voting systems fulfill different properties. In addition, even if certain voting systems do not always guarantee the fulfillment of certain properties, their likelihoods of achieving desirable results are significantly different. Therefore, although normative discussions are always related to different choices of values, the comparative analysis of the performance of different voting systems should make some of them more desirable (or less undesirable) than others. For example, the probability that the winner is the Condorcet-loser or that it is dependent on irrelevant alternatives is significantly higher with plurality rule than with any other system. According to these criteria, the systems based on exhaustive binary comparisons of candidates perform relatively well.

4.7 Some sports practices

In order to estimate the performance of different voting systems, for some time social choice theorists used mathematical calculations and computer simulations under the assumption that all possible voters' preferences are equally probable (an assumption usually called "random society" or "impartial culture"). This assumption has been criticized because it maximizes the probability of majority cycles and paradoxes, as for each preference order it presumes a symmetric or exactly opposed preference order able to prevent a stable majority. A better performance in which cycles tend to disappear has been stipulated on the basis of other assumptions regarding voters' preferences, such as, for example, the one assuming that there is a small group in the electorate with homogeneous preferences [41].

However, to estimate actual performances of certain voting systems, such as those based on exhaustive binary comparisons of candidates as proposed by Ramon Llull, Nicolas Condorcet and Arthur Copeland along the centuries, is difficult because some of them have been little used in real elections. Here I propose another approach which involves looking at the use of very similar rules in a number of modern sports tournaments. Just to take a major example, most state-level football championships in European countries are regulated by this kind of rules: exhaustive binary "comparisons" or matches between all competing teams are cast, different numbers of points are given for wins, draws and losses, and the winner is the candidate winning the greatest number of points.

In order to see whether these uses can enlighten us as to real performances of the above-mentioned procedures, I looked at rules and results in two outstanding football championships, the English premier league and the Spanish league of 1st division, from 1950 to 2010 (which therefore involves 120 tournaments with a total of 8,464 binary "comparisons" or matches). No Condorcet-winner capable of winning all matches has ever existed in these (and presumably other) competitions. In fact, thus, it is the Llull-Copeland system that is basically applied (variants include two or three points for a win, one point for a draw and no points for a loss).

As can be seen in Table 4.3, the number of draws in points is relatively high, 7% in England and 10% in Spain, so making the above-discussed problem of breaking ties a relevant issue. In these and similar competitions, draws in points based on matches are broken by counting goals (specifically by "goal difference" in favor and against, or by "goal average" in some previous periods). This additional procedure may evoke the proposal of counting votes when binary comparisons are inefficacious in producing a single winner, or the proposal of adopting the Condorcet procedure and the Cusanus-Borda procedure as a supplement when the former does not produce a winner, as suggested by Duncan Black [2].

The final column in Table 4.3 is based only on goal differences, which may produce comparable results to those that could be obtained with rank-count procedures such as the one proposed by Cusanus and Borda discussed above. Data for the two football tournaments show that if this procedure were not a supplementary one for the case of ties but the basic rule, in a very significant proportion of cases, around 21% in total, it would have produced a different winner than the winner with the procedures based on binary comparisons that are actually enforced. In other words,

Table 4.3: Binary and Goal-count Procedures in Football Tournaments, 1950-2010.

	Enforced rules (Llull-Copeland) Ties	Potential alternative winners (Cusanus-Borda) Goals-difference
England: Premier league	7%	18%
Spain: La Liga	10%	23%
Total	8%	21%

Source: Author's calculations with data for 60 annual tournaments in England and 60 annual tournaments in Spain for the 1950-2010 periods, involving 8,464 binary matches, from http://www.footballstatisticsresults.co.uk and http://www.futbolinSpain.com.

more than one fifth of the winners by rules based on binary comparisons would not have been winners (*caeteris paribus*) by rules based only on numbers of goals.

These data, although not focused on real voting and elections, may support the relevance of two points made in this article. First, the problem of ties in Ramon Llull's and similar systems based on binary comparisons of alternatives is relevant. This problem has grounded the above-presented hypothesis that Llull's innovative proposal of an eliminatory procedure in his third piece, DAE, could be motivated by troubles in proposing acceptable rules for breaking ties. Second, the differences between results by binary comparison methods and by rank-order counts or order procedures taking into account the absolute numbers of votes can be significant.

4.8 Conclusion

In this paper several voting systems and procedures proposed by Ramon Llull in the 13th century have been examined in their historical context and in the light of modern social choice theory.

Llull's basic proposal is a system based on binary comparisons of candidates by which the winner is the candidate winning by majority in the greatest number of comparisons. This system turns out to be more efficacious in preventing cycles and producing a winner than the celebrated Condorcet system requiring the winner to win all binary comparisons, although only for elections with five or more candidates.

The main points of this paper regard, first, the importance of Llull's contributions and discussion in the historical context of elections in the Christian Church and the emergence of majority rule as a new general principle for making enforceable collective decisions in replacement of traditional unanimity rule requirements. Second, in contrast to some previous tentative suggestions, careful reading of Llull's texts in their Catalan and Latin versions demonstrates that he did not propose a rank-order count system, such as those proposed later on by Cusanus and Borda. Third, a new hypothesis has been presented to explain Llull's later proposal of an eliminatory system of partial binary comparisons, or "successive procedure", which is based on the relatively high frequency of ties produced with exhaustive binary comparisons and the condemnation of lots by the Church in Llull's times. This paper has also shed some light on Llull's discussion of secret and open voting and on some applications of binary comparison methods in modern sports tournaments.

These analyses may give support to the statement that Ramon Llull should be considered the founding father of voting theory and social choice theory. Computer science could contribute substantively to making Llull's voting system of exhaustive binary comparisons applicable and efficacious in real elections in at least two ways: by designing mechanisms capable of reducing the costs of voting in multiple rounds, and by restoring conventional methods of breaking ties.

Table 4.4: Comparing Voting Systems

Voting procedures:	Plurality	Runoff	Approval	Llull	Condorcet	Eliminatory	Borda	Bentham
Desirable properties :				pairwise-comparisons			rank count	cardinal
Always the majority first preference	YES	YES	NO	YES	YES	YES	NO	NO
Never the Majority least preference	NO	YES	NO	YES	YES	YES	YES	NO
Always the Condorcet winner if it exists	NO	NO	NO	YES	YES	YES	NO	NO
Never the Condorcet loser	NO	YES	NO	YES	YES	YES	YES	NO
Independent on irrelevant alternatives	NO	NO	NO	NO	NO	NO	NO	YES
Monotonicity with voters’ preference changes	YES	NO	YES	YES	YES	YES	YES	YES

Bibliography

[1] Kenneth Arrow. *Social Choice and Individual Values (2nd ed.)*. New York, Wiley, [1951] 1963.

[2] Duncan Black. *The Theory of Committees and Elections*. Cambridge, Cambridge University Press, 1958.

[3] Anthony Bonner. *The Art and Logic of Ramon Llull*. Leiden, Brill Academic Publishers, 2007.

[4] J. S. Coggins and C. F. Perali. *64% Majority Rule in Ducal Venice: Voting for the Doge*. *Public Choice*, 97, 709–723, 1998.

[5] Josep M. Colomer. *Political Institutions. Democracy and social choice*. Oxford, Oxford University Press, 2001.

[6] Josep M. Colomer (ed.). *Handbook of Electoral System Choice*. Houndmills, Palgrave-Macmillan, 2004.

[7] Josep M. Colomer. *The Science of Politics*. New York, Oxford University Press, 2010.

[8] Josep M. Colomer. *City-republic*. In The Encyclopedia of Political Science, Washington DC, Congressional Quarterly and Thousand Oaks, CA, Sage, 2011.

[9] Josep M. Colomer. *Election by Lots*. In International Encyclopedia of Political Science. Thousand Oaks, CA, Sage, 2011.

[10] Josep M. Colomer. *Social Choice Theory*. In International Encyclopedia of Political Science. Thousand Oaks, CA, Sage, 2011.

[11] Josep M. Colomer and Iain McLean. *Electing Popes: Approval Balloting and Qualified-Majority Rule*. *Journal of Interdisciplinary History*, 29: 1, 1–22, 1998.

[12] Josep M. Colomer, Iain McLean and Haidee Lorrey. *Voting in the Medieval Papacy and Religious Orders*. In V. Torra, Y. Narukans, and Y. Yoshida (eds.). Modelling Decisions for Artificial Intelligence, Berlin and New York, Springer, 2007.

[13] Josep M. Colomer, Iain McLean and Haidee Lorrey. *Social Choice in Medieval Europe*. *Electronic Journal for History of Probability and Statistics*, 2007.

[14] Jean-Antoine-Nicolas de Caritat, Marquis de Condorcet. *Essai sur l'application de l'analyse à la probabilité des decisions rendues à la pluralité des vois.*. Paris: Imprimerie royale, 1785. (English translation in McLean and Urken 1995).

[15] Arthur H. Copeland. *A 'Reasonable' Social Welfare Function*. University of Michigan, Seminar on Applications of Mathematics to the Social Sciences, 1951.

[16] Josep Coroleu Inglada and Josep Pella Forgas. *Las Cortes catalanas: estudio juríidico y comparativo*. Valencia: Libreria Paris, [1876] 1993.

[17] M. Drton, G. Hägele, D. Haneberg, F. Pukelsheim and W. Reif. *A Rediscovered Llull Tract and the Augsburg Web Edition of Llull's Electoral Writings*. Le Médiéviste et l'ordinateur, 43, 2004. Available at: http://lemo.irht.cnrs.fr/43/43-06.htm.

[18] Dan S. Felsenthal and Moshe Machover. *After Two centuries, Should Condorcet's Voting Procedure Be Implemented?*. *Behavioral Science*, 37:4, 250–280, 1992.

[19] Víctor Ferro. *El dret públic català. Les institucions a Catalunya fins al Decret de Nova Planta*. Vic, Eumo, 1987.

[20] Salvador Galmés and Miquel Ferrà (eds.). *Obres de Ramon Llull. Libre de Blanquerna*. Palma de Mallorca, 1914.

[21] Martin Gardner. *Logic Machines and Diagrams*. 2nd ed. Brighton, Harvester, 1982.

[22] Allan Gibbard. *Manipulation of Voting Schemes: A General Result*. *Econometrica*, 41:4, 587–601, 1973.

[23] Gregory IX. *Decretalium compilatio*. In Corpus Iuris Canonici (Aemilius Friedberg ed.), Graz, Akademische Druck- u. Verlangsanstalt, [1234] 1959.

[24] Gregory X. *VI Decretalium*. In Concilium Oecumenicorum Decreta (Giuseppe Alberigo ed.), Bologna, Dehoniane, [1276] 1991.

[25] G. Hägele and F. Pukelsheim. *Llull's Writings on Electoral Systems*. *Studia Lulliana*, 41, 3–38, 2001. Available at: www.uni-augsburg.de/pukelsheim/2001a.html

[26] John Gilbert Heinberg. *History of the Majority Principle*. *American Political Science Review*, 20:1, 52–68, 1926.

[27] Martin Honecker. *Lullus-Handschriften aus dem Besitz des Kardinals Nikolaus von Cues. Nebst einer Beschreiburng der Lullustexte in Trier und einem Anhang über den wiederaufgefundenen Traktat De arte electionis*. Spanische Forschungen der Görresgesellschaft, Erste Reihe, 5, 259–239, 1937.

[28] Margi Lines. *Approval Voting and Strategy Analysis: A Venetian Example*. *Theory and Decision*, 20, 155–186, 1986.

[29] Jack Lively and John Rees (eds.). *Utilitarian Logic and Politics*. Oxford, Clarendon Press, 1978.

[30] Robert Howard Lord. *The Parliaments of the Middle Ages and the Early Modern Period*. *Catholic Historical Review*, 16:2, 125–144, 1930.

[31] Kenneth O. May. *A Set of Independent, Necessary and Sufficient Conditions for Simple Majority Decisions*. *Econometrica*, 20, 680–684, 1952.

[32] Iain McLean. *The Borda and Condorcet Principles: Three medieval applications*. *Social Choice and Welfare*, 7, 99–108, 1990.

[33] Iain McLean and John London. *Ramon Llull and the Theory of Voting*. *Studia Lulliana*, 32, 21–37, 1992.

[34] Iain McLean and Arnold B. Urken (eds.). *Classics of Social Choice*. Ann Arbor, University of Michigan Press, 1995.

[35] Samuel Merrill III. *Making Multicandidate Elections More Democratic*. Princeton, Princeton University Press, 1988.

[36] Léo Moulin. *Les origines religieuses des techniques électorales et délibératives modernes*. *Revue Internationale d'Histoire Politique et Constitutionelle*, 3, 106–148, 1953.

[37] Miranda Mowbray and Dieter Gollmann. *Electing the Doge of Venice: analysis of a 13$^{\text{th}}$-Century protocol*. Bristol, HP Laboratories, 2007. Available at http://www.hpl.hp.com/techreports/2007/HPL-2007-28R1.pdf.

[38] Hannu Nurmi. *Comparing Voting Systems*. Dordrecht: Kluwer, 1987.

[39] Llorenç Pérez Martínez. *El Ars notandi y el Ars electionis, dos obras desconocidas de Ramon Llull*. *Estudios Lulianos*, 3, 275–278, 1959.

[40] Bjorn Erik Rasch. *Parliamentary Voting Procedures*. In Herbert Döring (ed.). Parliaments and Majority Rule in Western Europe. New York, St. Martin's Press, 1995.

[41] Michel Regenwetter, Bernard Grofman, A. A. J. Marley, and Ilia Tsetlin. *Behavioral Social Choice*. Cambridge, Cambridge University Press, 2006.

[42] J. T. Richelson. *A Comparative Analysis of Social Choice Functions, I, II, III: A Summary*. *Behavioral Science*, 24, 355, 1979.

[43] Edoardo Ruffini Avondo. *Il principio maggioritario nella storia del Diritto Canonico*. Archivio Giuridico Filippo Serafini, IX, 1925.

[44] Edoardo Ruffini Avondo. *Il principio maggioritario*. Profilo storico, Milan, Adelphi, [1927] 1976.

[45] Mark A. Satterthwaite. *Strategy-proofness and Arrow's Conditions: Existence and Correspondence Theorems for Voting Procedures and Social Welfare Functions*. *Journal of Economic Theory*, 10, 187–217, 1975.

[46] Donald G. Saari and Vincent R. Merlin. *The Copeland Method. I: Relationships and the dictionary*. *Journal of Economic Theory*, 8, 51–76, 1996.

[47] Donald G. Saari and Vincent R. Merlin. *Copeland Method II: Manipulation, Monotonicity, and Paradoxes*. *Journal of Economic Theory*, 72, 148–172, 1997.

[48] Sara L. Uckelman and Joel Uckelman. *Strategy and Manipulation in Medieval Elections*. Accessed in http://ccc.cs.uni-duesseldorf.de/COMSOC-2010/papers/logiccc-uckelman.pdf, 2010.

5 Llull's Writings on Elections from the Perspective of Today's Research in Social Choice: an Economist's Viewpoint

Salvador Barberà

5.1 Introduction

Ramon Llull's known writings on the elections of church officials in clerical communities actually contain the proposals of different voting methods. His proposed methods are still today at the center of debates in social choice theory, as they represent plausible alternative solutions to fundamental dilemmas that any society faces when trying to design a voting system. In this note I will explain why even today there are sound theoretical reasons to defend any of these methods. Hence, the fact that Llull proposed several methods, each one of which is still very much alive in present debates, can be rationalized as one more proof of his genius and perceptiveness, rather than viewed as any form of inconsistency or confusion.

I will first briefly present Llull's proposals. Then I will discuss two main approaches that one can take regarding the role of voting, and describe how these two approaches became successively integrated into the main body of economic analysis. For each one of these approaches, I will then try to explain what are still some of the challenges that any designer in charge of devising a voting system has to meet, and how each one of the methods proposed by Llull is a strong candidate to overcome some of these challenges. In doing so, I will provide some references to contemporary literature where his or similar methods are proposed, in order to show this meeting between old and present concerns.[1]

5.2 Llull's proposals of voting methods

Hägele and Pukelsheim [3] provide the original versions of three texts written by Llull on elections, along with their English translations. Two of them are in Latin: *Artificium electionis personarum* (AEP) and *De arte electionis* (DAE). The latter is dated in 1299, the former is assumed to correspond to an earlier date, probably in the last part of the 1270s. The third text is part of his novel *Blaquerna* (B), written in Catalan during the 1280s, and is entitled *En qual manera Natana fou electa abadesa* (How did Natana get elected to be abbess). McLean and Urken also provide excerpts in English of *Blaquerna* and of *De arte electionis*.[2]

All the methods proposed by Llull are based on pairwise voting. In two of his writings, AEP and B, all pairwise votes are recorded. Clearly, in AEP, the proposed winner of the election is the candidate that has the largest number of wins in all the pairwise contests.[3] This is the method that we know today as the Copeland rule.[4] In B, the wording about how to find the winner is more ambiguous, and it may be given two interpretations: one is, again, that he proposes to choose the alternative with the most pairwise wins, as in AEP. This is the position endorsed by Hägele and Pukelsheim. Another reading, that McLean and Urken present as being quite plausible as well, would entail choosing the alternative that was given the maximum number of votes across all pairwise comparisons. This then amounts to a proposal of what we now know as the Borda rule.[5] As for his

[1] My presentation will be necessarily sketchy. The interested reader is referred to [3], to [4] or to the original works.

[2] Exact references to Llull's original works are found in the above mentioned references.

[3] A tie breaking procedure may be needed, and Llull also writes on this. We skip this aspect to concentrate on the basics.

[4] For a description and analysis of different rules, and a lucid presentation of axiomatic analysis, see [5, Part IV].

[5] I am not a historian and have a hard time when it comes to elicit what the author of a thirteenth-century text meant, once his expression becomes obscure to the contemporary reader. My personal reading oscillates between one interpretation of (B) and the other.

third proposal in DAE, it is clearly a different one, as it consists in selecting the final winner of a sequential elimination procedure based on pairwise votes, where this time the losing contender at each round of vote is definitely eliminated. This sequential method of voting is frequently used today in parliamentary practice.

In summary, Llull did propose at least two, and maybe even three different methods of vote: the one we call today Copeland's method, a sequential method of successive elimination, and maybe what we call now the Borda rule. As we can see, all of these methods are still studied nowadays, and eventually used in practice. Let us see why they are so important, even today.

5.3 Two main approaches to voting theory

Why do we need to vote? There are two basic and different approaches to answer this question. One is to assume that all members of society share a common goal, like the discovery of truth, but differ in their appreciation of reality: they may have different opinions about what is true, based on different experiences, partial pieces of information, etc ... Voting is then a way to search for the truth, by letting people express their opinions. A good voting method is one that combines people's opinions and reaches a correct decision about what the truth is. When it comes to electing people, the relevant truth is to elicit what candidate is best, or, as usually phrased in Llull's time, what candidate would be God's choice.

A second approach starts from a radically opposed perspective. There is no extrinsic definition of truth, or of what alternative is best. Each member of society has preferences over candidates, and is entitled to have this opinion count when making a social decision. Voting is then a method to aggregate individual preferences, or opinions, about the candidates into a collective preference, and the notion of what is collectively best should be the result of this aggregation process, rather than a starting point.

This second approach is the one proposed by Arrow in his seminal book *Social Choice and Individual Values* [1], and has been for years the reference for further research in social choice theory, extending to general formulations like, for example, those related to implementation theory.

More recently, however, economists have also re-discovered the importance of the first approach, considering voting as a method to share and to aggregate information. The revival of this point of view started with an increasing awareness of the importance of Condorcet's writings on jury decisions, expressed in terms of what is known as Condorcet's Jury Theorem. See Young (1988) for a good exposition.[6]

The distinction between the two approaches is relevant because each one of them focuses attention on different features of voting rules, and relies on different tools to analyze them.

Arrow's approach concentrates on the regularities of voting methods and relies heavily on their axiomatic analysis. Given a set of alternatives and a set of voters, a social state, or preference profile, is given by a specification of preferences for each voter over the different alternatives. A voting rule is then a function that assigns a social opinion to each admissible preference profile.[7] Social choice theory is devoted to the analysis of the regularities of voting rules, in search of particular methods satisfying different desirability criteria that can be expressed in the form of axioms. In the case of two alternatives, May's early characterization of the simple majority rule as the only one satisfying the attractive properties of neutrality, anonymity, and positive respon-

[6]As McLean and Urken point out, the history of social choice as a discipline is discontinuous, with frequent rediscoveries of topics and results that were forgotten for generations, even centuries. Economists started paying attention to it after Arrow's seminal work, and it is only much more recently that work on voting as information aggregation has become an active research topic. The perspective from other disciplines that converge to a common interest for voting methods may be somewhat different, but I think that this basic feeling of cyclical interest and forgetfulness is rather pervasive. Even today, when Condorcet's work starts being known, the contributions by medieval authors, and by Llull in particular, are relatively unknown.

[7]My description is deliberately vague here about the meaning of an opinion: it could be given by the choice of one or more alternatives, to be interpreted as those to be deemed as socially best, or, as in Arrow's formulation of "social welfare functions", by a ranking of all alternatives. Likewise, the term "admissible preferences" is meant here to signal that much of social choice theory is devoted to evaluate the performance of voting rules under specific subclasses of preferences (single-peaked, additive, separable, etc.), while in other cases no restrictions are imposed on the domains of these functions (then we talk about the universal domain).

siveness singles out this method as a prominent reference point. But problems arise when there are more than two alternatives. Arrow's Impossibility Theorem expresses the incompatibility of a number of axioms which, taken one by one, seem to capture desirable features that a voting rule should satisfy. Relaxing some of these conditions and finding combinations of compatible requirements that could be met by some voting rules has been an important part of the trade for social choice theorists. Among the most accomplished results in this literature one can find the characterizations of certain salient rules, in terms of their properties. We will come back to some of these results in connection with Llull's writings.

Let us now return to what we labeled as the first approach to voting, and comment on the methodological issues involved there. To be specific, let us return to simple majority voting, and to the case where there are only two alternatives. As I said before, this method's virtues can be made explicit through an axiomatic analysis. Condorcet's approach provides an alternative justification. Consider the case of a jury that must decide whether a defendant is guilty or not guilty. Here is a case where one can think that there exists an objective truth, a correct verdict, even if this truth may be unknown. It is also quite natural to assume that agents will share a common goal: they all would like the truth to prevail. Yet, their opinions on the case may differ, because of their distinct appreciation of the case. In modern terms, they may have received different signals regarding the true value of the parameter that determines whether the defendant is guilty or not. Then, the question arises whether the voting method to be used by a jury has consequences over the likelihood of making the good decision. Condorcet's jury theorem establishes that under specific and plausible conditions, adopting the decision recommended by a simple majority of the jury is the method that maximizes the probability of the correct decision. Again, this result is a starting point rather than the end of research. Further questions arise from it and new studies are called for, that even today constitute important lines of research. Can the result be extended to the case where society faces more than two alternatives? What if we prefer to guarantee that the innocent is not convicted, even at the cost of some mistakes in the opposite direction? Is the use of unanimity, rather than majority, a sure way to minimize the probability of convicting the innocent? Does deliberation play a role in the decisions of juries, and if so, what voting rules favor a richer use of the exchange of information that might arise from it?

Without getting into any detail, I wanted to emphasize here that, even if simple results, as the saliency of simple majority when only two options are at stake, do arise from both approaches, the natural setup for each one of them, and the kind of methods to be used, are quite different. And yet, Llull's proposals, both of them, stand up today as very prominent voting rules, whose relevance is sustained whatever of the two approaches we may take when analyzing voting methods.

5.4 Llull's proposed methods from the axiomatic perspective

As we already said, Llull's methods were rediscovered centuries later, and they are now in essence what we call today the Copeland method, voting by successive elimination and (probably) Borda's rule. What are the essential differences between these rules, from an axiomatic point of view?

Let us first recall that, when there are only two alternatives to choose from, the simple majority rule provides a rather satisfactory way to aggregate preferences, as expressed by May's characterization. But problems may arise when simple majority is applied in order to choose among more than two alternatives. Ideally, a choice based on majority should select an alternative that is not defeated by any other in pairwise contests. Such an alternative, if it exists, is called a Condorcet alternative. But there may be no such thing, because majorities may be cyclical and all alternatives may be defeated at some point. Even then, one can still require the following property, called Condorcet consistency: the rule should choose a Condorcet winner whenever one exists. And here is a first criterion to distinguish between the two methods proposed by Llull. What we now know as Copeland's, as well as the successive elimination method, do meet this property, while the Borda count violates it.

However, Llull's proposed rules satisfying Condorcet consistency, and in fact any method satisfying this requiremnt, will be unable to meet other attractive requirements that the Borda method will satisfy.[8]

One such property is called reinforcement, and requires the following. Suppose that two disjoint groups of voters M and M' face the same set of candidates, and that they both would select the same alternative a when using the rule. Then, the grand society resulting from merging both groups should still choose a. It turns out that the Borda rule (and its relatives) always satisfy this property, while no Condorcet consistent rule can guarantee that it holds. Another property is called participation, and requires the following. Say that candidate a would be elected under the rule by society M. Now consider an additional voter, who joins society and also participates in the vote. Then, either a should still be elected, or some alternative that the additional voter prefers to a. Notice that a violation of this requirement would lead agents to refrain from participating in the vote, as their actions could bring in a worse result than the one they could get by abstaining. And, again, Borda satisfies participation, while no Condorcet consistent rule can meet this requirement (if there are more than three alternatives).

Hence, the reader can see that the tension between properties held by the two methods proposed by Llull is unavoidable in the axiomatic setting. One has to choose between different kinds of axioms that will leave his proposals on different sides of the fence.

Let us now turn to examine Llull's rules in terms of the alternative view of voting as a method to elicit the truth. When we face two alternatives in a deterministic world, asking what is the best alternative is the same as inquiring what is the ranking of the two. But when we go to a world of uncertainty, and where more than two candidates are possible, formulating what is the best approximation to truth becomes tricky. What is the truth we are after? Is it the true ranking of alternatives, or is it finding out what alternative is actually best? The question is not rhetorical, because the best method to elicit the truth will not be the same under one circumstance than under the other. Specifically, under plausible assumptions, Borda's method will be the best to elicit

[8]Notice that the properties we shall now define have a different flavor than Condorcet consistency, as they apply to rules defined on variable populations.

what alternative is most likely to be the best. Whereas the best method to find out which ranking is most likely true will be a Condorcet consistent method (although not any of the two proposed by Llull). In other terms: the best candidate in the most likely ranking needs not be the most likely best candidate. And Llull's methods are, again, on both sides of this new fence. For the same society, one or the other may be more adequate, depending of what truth we may be after.

There is more in Llull's writing than just the general proposals I have discussed. He was also concerned about the choice of the choosers. He was aware of the danger that some voters might manipulate the procedure to their benefit (thus leaving room for a notion of private interest beyond the common interest).[9] He discussed methods to break ties. He distinguished between elections where votes are taken in public and those where ballots are secret. He anticipated that individual votes must already reflect the valuation of candidates in several directions. But I will stop here.

Of course, I am not claiming that Llull could have anticipated all the subtleties now studied by social choice theory, and that it took centuries to finesse. But I hope that his premonitory powers will amaze the reader as much as they amaze me, regardless of whatever reasoning may have guided his writings at each point.

[9] On that point, see [2].

Bibliography

[1] K. J. Arrow. *Social Choice and Individual Values,* 2nd edition. New York, John Wiley, [1951] 1963.

[2] S. Barberà. *Strategy-Proof Social Choice*. In K. J. Arrow, A. K. Sen and K. Suzumura (Eds.), Handbook of Social Choice and Welfare, Volume 2, Amsterdam, North-Holland, 2010.

[3] G. Hägele and F. Pukelsheim. Llull's Writings on Electoral Systems. *Studia Lulliana* 41, 3–38, 2001.

[4] I. McLean and A.B. Urken. *Classics of Social Choice*. Ann Arbor, University of Michigan Press, 1995.

[5] H. Moulin. *Axioms of Cooperative Decision Making*. Cambridge University Press, 1988.

[6] P. Young. Condorcet's Theory of Voting. *American Political Science Review*, 82, 4, 1231–1244, 1988.

6 Comments on the Logic and Rhetoric of Ramon Llull

Eduard Bonet

6.1 Introduction

Ramon Llull influenced the ideas of Leibniz (1646-1716) as to the possibility of building a universal language which would be calculable and could settle any kind of debate with mathematical operations. Ramon Llull is also considered a precursor of modern logical studies and computing machines. Following this line of thought I will briefly comment on his contributions to logic in relation to the classical tradition. But it is also important to emphasize that his mind-set is different from the ways of thinking of modern philosophers and scientists. For this reason, I will introduce some basic considerations as to his motivation for studying logic, his ways of combining logic and rhetoric and some aspects of his work that do not belong to the realm of science. My presentation is not based on direct research of his original texts, but is the outcome of the general reflections of a lay reader of some of his works.

The case of Ramon Llull, whose contributions to science were not based on scientific ideas, is not unique in the history of thought. It is worth recalling, for instance, that Johannes Kepler (1571-1630), one of the founding fathers of modern physics, had a religious and magical view of the universe, inspired by "the divine Pythagoras". He believed that the ratios of the distances between planets are related in such a way that the movement of these celestial bodies produces a universal harmony, a music he was able to hear with his acute capacity of perception. He also believed that the planets move around the sun because it is the visible symbol of God in the universe. I do not mean that we can establish significant similarities between Ramon Llull and Johannes Kepler, but the moral of this fable is that we have to evaluate the logic of Ramon Llull independently from his religious motivations.

6.2 The ways of God

Let us follow the life of Ramon Llull. Mallorca was conquered by king Jaume I of Aragon and Catalonia from Muslim domination in 1229 and became a place of political submission, intercultural frictions and possible intellectual encounters. The conquest was the first step in the commercial expansion, political influence and linguistic predominance of Catalonia all around the Mediterranean Sea. We are now in the year 1262: Ramon Llull is thirty years old and married; his father came to Mallorca with the king and acquired responsibilities in the Royal Court. One night, Llull is in his bedroom writing a passionate letter to a mistress, when suddenly God appears to him nailed to the cross and speaks to him. This is the first illumination in which God asks Llull to devote his life to the conversion of infidels to Christianity. Ramon Llull obeys the divine mandate, leaves everything he has and travels extensively preaching to Muslims and Jews.

Ramon Llull leads an extremely active life. He prepares himself and learns for his mission; he writes a large number of books on all kinds of cultural subjects in Catalan or Latin, such as knighthood and the education of young people, which he relates to moral principles for his apostolate. He translates and re-translates his texts into two or three languages. Ramon Llull travels a lot and preaches in many places; he tries to persuade popes, kings and knights of the value of his projects; he even survives a shipwreck.

In the long and rich experience of Ramon Llull, there is another extraordinary event which we will look at with interest. In 1274, when Ramon Llull is already forty-two years old, he has a second illumination at mount Randa. This time God gives him some general ideas about the content and form of his books. Their content has to be based on the Christian faith, their arguments have to prove the truth of this faith and their style has to include the oriental forms favoured by infidels in order to convince them. Ramon Llull believes that converting infidels requires the sword on one side and the word on the other.

I find that it is absolutely extraordinary that Llull, with regard to the second illumination, insists on logical proofs and rhetorical means of persuasion. It is clear that the interest of Ramon Llull in logic and rhetoric was motivated by the apostolic activities that he undertook by the direct mandate of God. Without overlooking this, we can, however, study his contributions to logic and rhetoric from a strictly academic point of view.

6.3 A long search for Knowledge, Logic and Rhetoric

A learning period. After the first illumination in 1262, Ramon Llull sells all his properties and devotes three years to religious pilgrimages, studies and conversations with a number of important scholars. In 1265, he re-thinks his strategy for preparing himself and goes back to Mallorca. During the next nine years (1265-1274) he studies theology and almost all the disciplines of his time, travels to many countries and tries to persuade popes and kings to give support to his apostolic projects. As he is progressing in his training, he writes books in which he expresses his ideas. In this way he begins to build his vision of the world, which involves an ontology centred on God and which will culminate in his *Ars magna*. In Mallorca, he learns Arabic from a slave and this knowledge enables him to talk with Muslims, to introduce new ideas into Western Culture, and to translate books into Catalan and Latin. This is the case of the following treatise on logic:

The *Logic of al-Ghazzali*. Ramon Llull translated the work from Arabic into Latin (*Compendium Logicae Algazelis*) and Catalan (*Lògica del Gatzell*); this work had a strong influence on the development of Llull's logical ideas.

Ramon Llull, besides this line of enquiry also explored new logical ideas. In the book *Llibre del gentil i dels tres savis* (*Book of the Gentile and the three Wise Men*), he introduced all the binary combinations of virtues and of virtues and vices. He developed this method in many other writings and integrated it into the *Ars magna*. I will comment on it, when dealing with his use of logic and rhetoric.

6.4 Re-thinking the arts and sciences

After the second illumination (1274), Ramon Llull worked on the project of re-modeling many arts and sciences, basing them on his ontology centred on God. The term *New* in the title of many books, such as *New Metaphysics* (*Metaphysica nova*) and *New Astronomy* (*Astronomia nova*) possibly reflects his purpose of innovation. In this presentation, I will emphasize the *New Rhetoric* (*Rhetorica nova*) and the *New Logic* (*Logica nova*).

New Rhetoric (Rhetorica Nova). Ramon Llull wrote this work in Catalan in 1301, while he was staying in Cyprus, but this version is lost. We have access, however, to its Latin version (see [1] which includes an introduction, the Latin version and an English translation). The *New Rhetoric* introduces many examples of preaching and from this point of view it can be considered a conceptual handbook for preparing sermons. In fact, after its publication, Ramon Llull wrote several books on this subject.

The *New Rhetoric* is the most original text on rhetoric of the Middle Ages. While in this period contemporary books followed Cicero's work *On Invention*, Llull divides his text into the following four parts: on the order of speech, on the beauty of speech, on knowledge about speaking and on love (*caritas*) in speaking. The emphasis Ramon Llull puts on virtues and love relates this book to new trends in rhetoric that should emerge at the end of the 20$^{\text{th}}$ century such as those fostered by Deirdre McCloskey [4].

New Logic (*Logica nova*) (1303) has a Latin and a Catalan version (*Lògica nova*); *Lògica nova* was critically edited by Antoni Bonner in 1998 and published by Patronat Ramon Llull, Palma de Mallorca. As the editor emphasizes, the text contains many elements of the Medieval heritage of Aristotelian logic, but in the new book some of these traditional topics are absent and new formulations included. To situate Llull's logic in the medieval tradition requires in-depth expertise in historical and philosophical research. Amazingly, the book begins with a Tree of Being, which deviates from Porphyry's classical tree, presenting many logical distinctions and describing the three classical figures of syllogism. Its final section applies these concepts to the sciences of law and medicine. From our point of view, it is very important to emphasize that *Lògica nova* makes no mention at all of the contributions to logic, such as the use of symbols, binary combination, and logical wheels, that Ramon Llull began to develop before he wrote the *New Logic*.

6.5 Ramon Llull's system of logic

Ars magna and Ars brevis. Ramon Llull presented his ultimate conceptual system, with his ontological principles and logical methods in the book *Ars magna*, or *Ars generalis ultima* (1308), which was written in Latin, and produced a shorter and more didactic version with the title *Ars brevis*, which he also finished in 1308. These books constitute basic references for understanding Ramon Llull's logic.

Concepts and symbols. The conceptual system of the *Ars magna* introduces the following six categories; each of these includes nine concepts, logical relationships or questions, of which we will mention a few examples:

First category:	Absolute Principles or Virtues of God such as Goodness, Greatness and Eternity.
Second category:	Relative Principles or Logical Relationships such as Equality, Majority and Minority.
Third category:	Subjects or objects of study such as God, Angels and Humans.
Fourth category:	Rules of questioning such as What?, Where? and When?
Fifth category:	Human Virtues such as Justice, Wisdom and Temperance.
Sixth category:	Human Vices such as Pride, Greed and Lust.

Each of the nine divine attributes, relationships, questions, etc. is represented by a capital letter from B to K, while A represents God. It is important to point out some properties of this symbolic language: first, each letter represents one concept in each category but, as we have six categories, it has six interpretations. This semantic polyvalence would be disastrous in modern symbolic languages and calculations, but it is not a hindrance for Ramon Llull, because he is always aware of the categories he is working with. Second, this form of coding allows him to work easily with combinations. Third, letters represent one concept and do not operate as "variables" in modern logic. This property emphasizes the fact that Ramon Llull's logic is a logic of content, not a formal logic as in the Aristotelian tradition. But, of course, it will foster new approaches towards formal modern logic and computers.

Combinations. The *Ars magna* introduces several systematic studies of binary and even ternary combinations of the concepts of one category, such as human virtues, or of two or more categories, such as human virtues and human vices. This approach was essential for Llull's strategy: Ramon Llull worked more with comparisons than with syllogistic deductions and the study of systematic comparisons was very important for him. Nowadays we have easy algorithms and concepts such as the Cartesian products of sets, for establishing combinations, but for him the set of ternary combinations was hard work.

The logical wheels. Ramon Llull's logical wheels, represented in Figure 6.1, constitute a singular precedent of the mechanical devices for logical computations that were introduced many centuries later. In this machine, each wheel can be associated with a category of concepts and it has the letters B, C, D, E, F, G, H, I, K which represent nine concepts of the corresponding category. In the initial position of the figure under the letter B of the superior circle, there is the letter B of the middle circle and under it the letter B in the inferior circle. In this way the machine displays the sequence B, B, B and another nine sequences. We can turn the middle wheel putting the letter C under the B and turn the inferior wheel putting the letter D under the C. In this way the machine displays the sequence B, C, D and another nine sequences. Turning the second and third circles to all positions, Ramon Llull managed 252 sequences of concepts without repetition and could make systematic comparisons.

A possible application of Ramon Llull's machine is to use it as a heuristic device for solving a problem related to Aristotelian logic: in the syllogism, from the premises "all C are D" and "all B are C", we deduce the conclusion "all B are D". In it, C is the middle term, D the major term and B the minor term. This is the classical way of deduction. But, sometimes we have the follow-

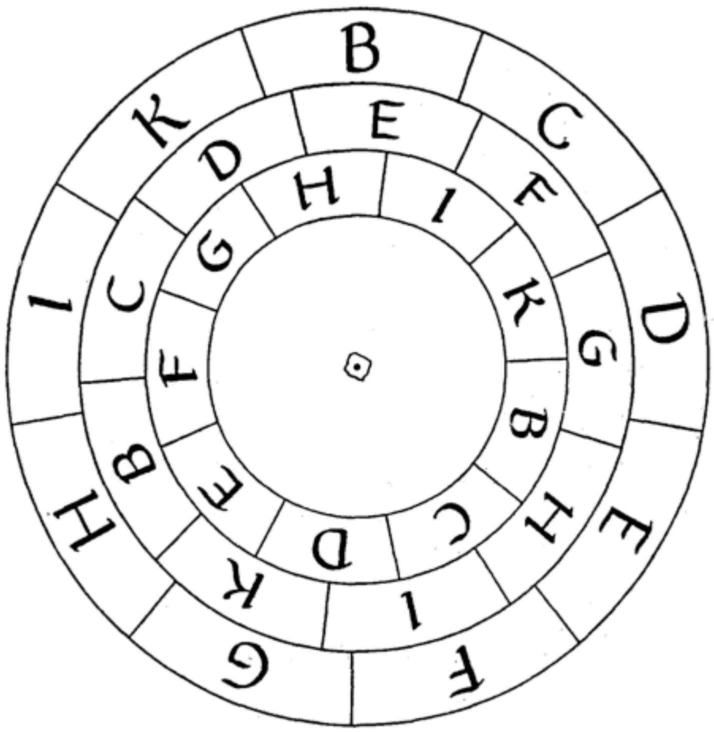

Figure 6.1: The Fourth Figure

ing problem: we try to prove that all B are D, and we look for a middle term C that allows us to introduce the true premises of the syllogism. In this situation, Ramon Llull's device can be used in the following way: we fix B in the inferior wheel and D in the superior wheel. Then, we turn the middle wheel to all its possible positions, examining them to see whether one of them solves our problem.

6.6 Logical and rhetorical strategies in Ramon Llull: A paradigmatic example

The Book of the Gentile and the Three Wise Men (*Llibre del gentil i els tres savis*). As the obsessive and passionate purpose of Ramon Llull was to convert the infidels, in his teaching and preaching activities he tried to keep the interest of his audiences on religious subjects, to prove that Christian faith is true and to persuade people of this truth. For this reason, he applied both his logical and rhetorical skills in his speeches and writings such as the *Book of the Gentile and the Three Wise Men* (an English translation is included in [2] and the Catalan version in [3]). Both texts are edited with commentaries by Bonner and constitute an outstanding example of the ways Ramon Llull combined logical proofs and rhetorical persuasion.

The book is about a Gentile from a remote country, who has never heard of God and resurrection, and about his distressing search for the meaning of his life. In a fortuitous encounter on a plain close to a forest, he meets three wise men, a Christian, a Jew and a Muslim who are arguing about the merits of their religions. The context facilitates the dialogue between them. For us, it is very interesting to focus on the logical and rhetorical strategies of the text.

- *Logical strategies*. The book introduces the following concepts:
  - Seven Divine Virtues: Goodness, Greatness, Eternity, Power, Wisdom, Love and Perfection.
  - Seven Human Virtues: Faith, Hope, Charity (Love),

Justice, Prudence, Fortitude (Courage) and Temperance (Self Control).

– Seven Vices (Capital or Mortal sins): Gluttony, Greed, Lust, Pride, Accidie, Envy and Wrath.

With these concepts, Ramon Llull introduces five sets of "combinations":

– First, the twenty-one binary combinations of Divine Virtues.

– Second, the forty-nine couples formed by a Divine Virtue and a Human Virtue (in our language this set is a Cartesian Product).

– Third, the forty-nine couples formed by a Divine Virtue and a Vice (Cartesian product).

– Fourth, the twenty-one binary combinations of Human Virtues.

– Fifth, the forty-nine couples formed by a Human Virtue and a Vice (Cartesian product).

Ramon Llull was very interested in comparing concepts and those five combinatory sets allowed him to undertake a systematic study. One of his aims was to prove the coherence of Divine Virtues (first set); the coherence of Divine Virtues and Human Virtues (second set), the coherence of Human Virtues (fourth set), the incoherence of Divine Virtues and Vices (third set) and the incoherence of Human Virtues and Vices (fifth set).

- *Rhetorical Strategies*. The *Book of the Gentile* is a philosophical and theological narrative influenced by Arabic tales, which introduces a female incarnation of Intelligence and some marvelous landscapes and events. It presents the sorrow and afflictions of the Gentile in his search and describes in short, efficient expressions the beauty and exuberance of the forest in which he lives for a while. The three wise men arrive at a plain with a fountain and five extraordinary trees, when a beautiful lady riding a horse joins them. At their request she explains that each of these five trees contains in its leaves the signs of the combinations of one of the five combinatorial sets that I have commented. This is the way Ramon Llull presents his logical concepts. Once the lady has left, the Gentile meets the wise men and they wish him God's help in his search. This is the first time that the Gentile hears about God and he asks that they introduce him to knowledge about Him.

6.7 Conclusions

An analytical approach allows us to study in separate fields logic, rhetoric and the doctrines of Ramon Llull. A synthetic approach introduces us to his thought. There are many ways of combining analysis and synthesis and the present contribution aims at offering an introduction to Ramon Llull that can be useful for research on logic and Artificial Intelligence. All of us apply logic in our programs and also apply rhetoric in persuading people of their merits.

Bibliography

[1] Ramon Llull. *Ramon Llull's New Rhetoric*. Mark Johnston, ed., Davis, CA, Hermagoras Press, 1994.

[2] Ramon Llull. *Selected Works of Ramon Llull*. Translated by Anthony Bonner, Princeton, Princeton University Press, 1985.

[3] Ramon Llull. *Obres Selectes*. Antoni Bonner, ed., Palma de Mallorca, Moll, 1989.

[4] Deirdre N. McCloskey. *The Bourgeois Virtues — Ethics for an Age of Commerce*. Chicago and London, Chicago University Press, 2007.

7 Some Features of the Semantics in the Lullian Tradition

Mauricio Beuchot

7.1 Introduction

In what follows I will try to collect some data on the semantics in Ramon Llull and in Lullism, which had its *floruit* during the mid to late Middle Ages, starting at the beginning of the early fourteenth century. At the time, the tradition of the so-called properties of terms already existed, as a part of the contemporary discourse on logic, although now it was applied to the parts of semiotics: syntax, semantics and pragmatics. Indeed, semiotics, defined as the general theory of sign, has three parts or dimensions; namely, syntax, which is the study of the relations of signs to each other; semantics, which studies the connection between signs and the designated objects; and pragmatics, which, in turn, studies the relations of signs with their users. Therefore, syntax studies the relationship of coherence; semantics, of correspondence; and pragmatics, of signs' usage.

In the Middle Ages, all these issues were addressed in the studies related to logic. There were the *parva logicalia*, also known as minor treatises on logic, which concerned the sign and signification; there were the logical properties of terms, which were addressed at the beginning of the manuals of logic (*summulae logicales*), precisely to deal with the question of the terms. This tradition, which studied language from the logic perspective, was coexistent with another tradition, more linguistic, which was the *grammatica speculativa*. As far as the Lullian school is concerned, these two traditions converge: the logical one and the grammatical, or rather the linguistic one; the aim of this paper is to demonstrate how they are approached.

7.2 Syntax and Semantics in Llull

Ramon Llull plays a prominent role in the history of logic and, therefore, in the history of philosophy of language. He is considered to be one of the forerunners of the perfect language ideal and of a general logical calculus used to achieve encyclopaedic or universal knowledge [1]. He knew how to combine the Greek, Arabic and Medieval Latin traditions in his research on logic and achieved results which only now begin to be truly appreciated [1].

Given his endeavour of an *Ars magna*, Llull was more interested in the combinatorial logic than in the properties of terms from the tradition of the *summulae* as those of Peter of Spain [13, pp. 67ff]. Rather than that, he centred himself on the key terms of his art and the combinations, as can be seen in the *Logica nova*, which does not tackle the issue of the logical and semantic properties of terms [9].

However, among his followers attention was paid to problems concerning language, commonly approached within the field of logic in that time, which latter had the function of applied semiotics. It is well known that the issues in that time addressed by logic are now treated by philosophy of language or by semiotics, namely the ones related to semantics. While discussing the matters related to the sign and term, the questions pertaining to the *parva logicalia*, or *proprietates logicae* of terms, were being exposed in detail; namely, the theory of signification, the supposition and allied issues, such as the amplification, restriction, appellation, alienation, diminution, etc. [3, pp. 29–34].

The signification (*significatio*) was a property of the term outside the proposition, and consisted of the acception of a term which conveyed a certain meaning; in other words, it is the capacity which the term has to mean something. It coincides with what we call, after Frege, the "sense". On the other hand, the supposition is a property which the term has only when it is within the proposition, in other words, in the propositional context. It can be said that it coincides with what we call the "reference", after Frege. In fact, it is the capacity to replace or be in the place of the meant thing. For example, the term "man" may mean something by itself, separately; however, it can only have supposition in the context of the proposition, as in "the man disputes". Moreover, the term had some other properties related to the supposition. Thus, amplification was precisely the expansion of the supposition in order to designate more meanings than it usually had. For example, when a past tense was used, as in "the man lived in these lands" (thus covering not only the present but also the past), or a future tense, as in "man will destroy the planet" (in this way the meaning is extended to those men who are about to come). The restriction made the term mean less than it usually meant, as in "the white man runs", as it relates to fewer individuals than if it only stated "the man". The other notions are: the appellation, which was the supposition of adjectives, alienation and some others. In the Lullian tradition, however, only supposition, amplification and restriction are treated, therefore they are the ones to be discussed here.

The theory of supposition concerned what is now called syntax and semantics, as it comprised syntactic elements, such as the control of the logical quantification of the statements, but also, and above all, it involved what is now called semantics, in other words, the theory of meaning, the manner of making reference to the objects.

The great work of Llull, which revolved around the logical construction of philosophy and other fields of knowledge —even as an unattainable ideal [6, pp. 65ff]— made him aware of the importance of a perfectly logical language. Therefore, he took great care over the logical syntax, although the semantics was studied by his followers, amongst them Bernard de Lavinheta, who addressed the issues of philosophy of language which were being treated in that time. Thus, together with other issues, the logical properties of terms were tackled.

Such is the case of *Dialectica seu logica nova*, which was attributed to Llull for a long time and was edited by Lavinheta. There, the suppositions are briefly presented, with their respective amplifications and restrictions.

They are referred to as the properties of terms, which was the common designation. There is no mention of signification (*significatio*), whereas supposition (*suppositio*), amplification (*ampliatio*) and restriction (*restrictio*) are discussed.

It is worth noticing that these issues were not studied at the beginning of the work, where the notion of term was discussed, but after having tackled the term and the proposition. This seems to be of interest, as the supposition is presented within the proposition, and not separately, as is the case of signification.

They are studied briefly, but manifestly. *Suppositio* is defined as "the acception of a term standing for a universal or singular thing".[1] It is noteworthy that the reference is made to universal things, although it is understandable in the context of Platonic-Augustinian realism in which the work is settled, as well as an important part of the Franciscan School (within which the realism imposed conceiving universals as certain things, in other words, as exemplary ideas in the divine mind) [12, p. 74]. Then the supposition is divided into simple, personal and material. The simple one would be making reference precisely to the universal things ("homo est species"); the personal one, to the individual things ("homo currit"), and the material one, to the term itself ("homo est dictio dissyllaba").

On the other hand, the amplification is another property of a term which allows it to be placed in different times. Even though the Lullian author understands this notion as usual, what is most noteworthy are the three rules which he mentions in order to apply it: "The first one is that in every proposition in which there is a verb in a past tense or a participle, the preceding term is extended with a view to what it is or what it was, as in 'the virgin was a prostitute'." The second rule [is that] in any proposition in which there is a verb or a future participle, the preceding term represents what it is or what it will be, as "the old man will be a boy". The third rule is that every term of the proposition which is related to the verb 'can', or its participle, represents what there is or what there may be, as in 'white can be black' [10, p. 152].

The restriction is —as it was usually understood— the representation of a term in a proposition which makes reference to fewer meanings than it would have according to its nature (without the restriction), such is the case of the term "man" combined with the term "white" in the proposition "every white man runs".

As one can appreciate, the discussion of the properties of terms in the *Dialectica* is very concise. Eventually, however, they are treated sufficiently enough to be able to prove that the Lullian au-

[1] [10, p. 152] (This work about logic is still on the list of Llull's works presented by Cruz Hernández [6, p. 370, no. 46]).

thor contributed to the vast tradition of the medieval logic of semantics as well as to the study of the (logical) properties of terms. Such is the proper semantics of the Lullian tradition.

7.3 Pragmatics and Rhetoric

What we might call the pragmatics of Ramon Llull is set out in his rhetoric. Thus, in the *Retòrica nova*, he studies the form, matter and purpose of words, which are double. Indeed, the word has a twofold form: the one which it has by itself and which is essential to it, and the other one which it obtains by being attached to another word, and it is accidental. For example, the word "regina" contains in itself the essential beauty, but some other beauty can be added to it when we say "regina est bona", as the goodness also contains its beauty. Obviously, this is very useful as far as rhetoric is concerned, as in rhetoric the beauty of words matters [11, p. 98].

Additionally, the matter of words is twofold: one is essential and the other one is accidental. The first one is contained in "regina", "magnitudo" and "pulchritudo" in: "Regina habet magnam pulchritudinem", because they make reference to beauty. The second one is contained in "regina", "magnitudo" and "bonitas" in: "Regina habet magnam bonitatem", as they also lead to beauty, which is next to goodness [11, p. 100].

Equally, the purpose of words is twofold: one is the expression or explanation, the other one is the one which the speaker has in mind and which is the reason why he speaks. The first one can be found in "The Queen, who is beautiful, is of great kindness". The second one is contained in, "for example, when a damsel says to the queen: 'Your majesty, you are of great beauty and great kindness', pursuing this purpose, namely, that her majesty finds her a husband; as this was the purpose with which the damsel spoke to her lady in such a way" [11, p. 102].

In the latter, in the words' purpose as presented by Llull, one can find the usage and the speaker's intention, similar to the "speaker's meaning", so characteristic of the pragmatics, and even bearing some resemblance to the distinction of Austin and Searle between constative and performative.

These examples provided by Llull and his school combine well with another medieval tradition of speculative grammar, the one of Thomas of Erfurt and the *modistae*, who were given this name because they studied *modi dicendi* in accordance with *modi cognoscendi* and *modi essendi*.[2] In other words, the manners of speech, of grammar or linguistics, were seen as related to the manners of knowing, epistemology or theory of knowledge, and also related to the manners of being, ontology. It is the tradition which is derived from the classical grammar, while the other one comes from logic. Both of them are typically medieval, although the logical one is more known.

7.4 The Ontological Basis of Semantics: the Universals

Llull also addresses the issues of epistemology of language as well as the treatment of the sixth sense (called *affatus*), used to account for the oral expression (*vox significativa*) [14]. The only point which I shall discuss here, as far as his ontology of language is concerned, is the problem of universals.[3] The problem involves establishing the nature of our concepts and universal terms (generic and specific), that is, whether they are a part of reality or just something in the mind; the answers to these question can be found either in realism or nominalism. Realism, in turn, can be divided into two: Platonic realism, within which there are abstract, archetypal ideas which correspond to the specific and particular things, and Aristotelian realism, according to which there are no prototypical ideas; on the contrary, universals are the forms of things settled in the matter, which provide our concepts with a reality-based foundation [4, pp. 439ff].

Llull supports, as did part of the Franciscan School, the Platonic-Augustinian realism similar to the one of Saint Bonaventure. Thus, he sees genus as something real, and enumerates five rea-

[2] The connection between Llull and the linguistic tradition of speculative grammar and the *modistae*, is demonstrated by Josep Batalla, Lluís Cabré and Marcel Ortín, in [11, p. 99, note 10 and p. 101, note 13].

[3] There are other, interesting, ontological subjects, e.g. the one of the categories. Cf. [8].

sons for it: (1) The genus *body* has to be real because if not, it could not be divided into animate and inanimate. (2) The nature abhors a vacuum, which would occur if the genus was not real. (3) If it was not real, the creation of the world would have been impossible. (4) If it was not real, the individuals would not suffer any influence of the heavenly bodies. (5) A given state, such as the one of the animality, must have a real entity which we call genus [7, p. 80].

Within this idea of universals another idea of his can be found, namely the one of the transcendent dignities, which has some Platonic-Augustinian features, very similar to the viewpoint of Saint Anselm, from whom he takes the idea of using evident reasons to demonstrate the contents of faith. Any solution given to the ontological problem of universals, has important consequences for the way philosophy of language is conceived. The concepts of meaning and reference will depend upon it, as can be seen in G. Frege, who, due to his Platonism, understood the meanings as abstract entities, and even accepted abstract entities as the references of general terms [5].

7.5 Conclusion

As we have seen, Lullism combines the two traditions of the medieval philosophy of language: one which is derived from logic, and is the most prominent one, and the other one, that of the *modistae* and the *grammatica speculativa*, which is also present. However, the tradition which is most vivid in Llull himself is the one which comes from logic, rather than the one from the traditional grammar, which is more linguistic. As a matter of fact, Llull's main concern was logic; it is not by chance that he is considered one of the most influential figures in the history of this discipline, the illustrious predecessor of symbolic logic or mathematical logic, because of his *Ars magna*. However for this endeavour he needed to study the language; and that was what he did as a basis of his remarkable construction.

Bibliography

[1] Mauricio Beuchot. *El ars magna de Lulio y el ars combinatoria de Leibniz*. Diánoia 31, 183–194, 1985.

[2] Mauricio Beuchot. *Prólogo a Juan José Ibarra de la Paz, O.F.M., La lógica simbólica en el Beato Ramón Llull y George Boole: del silogismo a la ciencia*. Zapopan (Mexico): Orden de Hermanos Menores — Provincia de los SS. Francisco y Santiago, pp. VII–VIII, 2010.

[3] Mauricio Beuchot. *La filosofía del lenguaje en la Edad Media*. Mexico: UNAM, (2nd ed.), 1991.

[4] Mauricio Beuchot. *El problema de los universales*. Mexico: UNAM, 1981, 1981.

[5] Mauricio Beuchot. *El problema de los universales en Gottlob Frege*. Crítica 9/26, 65–89, 1977.

[6] Miguel Cruz Hernández. *El pensamiento de Ramón Llull*. Valencia: Fundación Juan March — Editorial Castalia, 1977.

[7] Anton Dumitriu. *History of Logic*. Tunbridge Wells, Kent, England: Abacus Press, vol. II, 1977.

[8] Alexander Fidora. *Peirce's Account of the Categories and Ramon Llull*. Studia Lulliana 47, 175–186, 2007.

[9] Raimundus Lullus. *Logica nova*. Ed. Walter Euler, Turnhout: Brepols, 1998.

[10] Raimundus Lullus. *Dialectica seu logica nova M. Raymundi Lullii, diligenter emendata: restitutis quae olim fuerant sublata: et additis Tractatu de onventione medii, ítem Tractatu de conversione*

subjecti et praedicati per medium: per M. Bernardum Lavinhetam. Opera, Argentorati, 1617.

[11] Ramon Llull. *Retòrica nova*. Ed. Josep Batalla, Lluís Cabré and Marcel Ortín, Turnhout: Brepols — Santa Coloma de Queralt: Obrador Edèndum, 2006.

[12] Sebastià Trías Mercant. *Raíces agustinianas en la filosofía del lenguaje de R. Llull*. Augustinus 21, 59–80, 1976.

[13] Peter of Spain. *Tractatus llamados después Summule Logicales*. translated by Mauricio Beuchot, México: Instituto de Investigaciones Filosóficas, UNAM, 1986.

[14] Elena Pistolesi. *Paraula és imatge de semblança de pensa. Origine, natura e sviluppo dell'affatus lulliano*. Studia Lulliana 36, 3–45, 1996.

8 Adaptive Reasoning in Ramon Llull's *Liber de syllogismis contradictoriis*

Guilherme Wyllie

It was in May 1308 when Llull started a process of revising, improving and elaborating certain innovative methods for argumentation. With a view to the works of some of the major theologians from the University of Paris, such as Thomas Aquinas, Giles of Rome, Martin Anglicus and Richard of Middleton [16, pp. 292–306], Llull regrets that

> some of the Christians who are highly esteemed for their knowledge —and this is both shameful and to be deplored— say that the holy Catholic faith is more improbable than probable. Thereby, great disbelief follows among the infidels, who for this reason believe that our faith is nullified; and occasionally some Christians are perversely suspicious about the faith. Moreover, some assert and hold that faith is true, yet that it seems not to be true from the point of view of reason. Thus, with the Holy Spirit's help, we shall try to prove as much as we can that such faith is both true and probable. [15, p. 328][1]

[1] "Aliqui christiani et magni in scientia nominati, quod pudendum et plorandum est, dicunt, quod fides sancta catholica est magis improbabilis quam probabilis. Vnde sequitur infamia magna apud infideles, qui ex hoc credunt fidem nostram esse nullam; et forte aliqui christiani contra dictam fidem sinistre suspicantur. Vlterius aliqui dicunt et asserunt, quod fides est uera, sed secundum modum intelligendi uidetur non esse uera. Vnde adiuuante sancto Spiritu conabimur, quantum poterimus, ad probandum, quod dicta fides sit et uera et probabilis."

On the one hand, Llull was highly concerned with the growing acceptance of the claim put forward by some Parisian masters from the Faculty of Theology, namely that the articles of faith could not be proved; on the other hand, he was outraged by the ongoing spread of Averroistic theses at the Faculty of Arts, some of whose members maintained that philosophy was totally independent from theology. In order to meet such challenges, Llull decided to leave Montpellier, where he was living by that time, and to move to Paris.[2] Yet, before leaving, he accomplished in 1308 a thorough intellectual preparatory phase which included not only a critical assessment of the current theories of scientific demonstration, but also the development of five new argumentative devices,[3] namely (i) the fallacy of contradiction; (ii) the discovery of the middle term; (iii) the proof by the superlative degree; (iv) the contradictory syllogisms and (v) the contradictory suppositions.[4]

From 1309 to 1311, Llull remained in Paris where he eventually achieved a considerable academic reputation [11, p. 557]. Actually, his *Ars*, like other of his writings (including some thirty works which he published during this period), was subjected to a detailed examination, as a result of which in February 1310 forty scholars and fellows from the Faculties of Medicine and Theology subscribed a letter of approval of his doctrines, right after attending one of his lectures (see [20, p. 302] and [8, pp. 108–112]).

A careful reading of Llull's writings from his third and last visit to Paris shows the progressive implementation of these new logical devices construed in 1308; among these, his contradictory-syllogisms-method is especially relevant, for it involves outstanding logical strategies. It was the result of manifold improvements, focusing on a technique based on the contraposition of rebutting arguments.[5] Such a method was formulated for the first time in May 1308 in the *Ars compendiosa Dei*, a Christian Theology manual

[2]On Llull's anti-Averroist campaign in Paris, see [9].

[3]On Llull's preparatory work for his campaign against Averroism in Paris, see especially [4].

[4]Readers will find an overview of the principal logical doctrines developed by Llull during this period in [5].

[5]In general, a rebutting argument is one whose conclusion is the negation of the conclusion of another argument, so that if an argument entails that 'p' is the case, then the corresponding rebutting argument asserts that 'not-p' is the case [3, p. 2]

aimed at providing guidance to missionary work. Throughout this extensive work, God's attributes, which Llull called *principia*, are considered each by means of an opposition of "two demonstrative syllogisms, primary, true and necessary; and two untrue syllogisms, such that from the untrue we can know the true ones and reciprocally".[6]

However, it seems that Llull gave up this method for some time and set himself to deploy other argumentation resources, until, in February 1311, he came back to the former method in his *Liber de syllogismis contradictoriis*. This anti-Averroistic work displays a dialogical structure; it is motivated by the story of a supposed dispute between a disciple of Llull's and an Averroist. In the prologue, Llull's disciple attributes forty-four theses to the Averroist, announcing that he will prove each of them to be false. Moving ahead, he sets out —in the first of the three distinctions of the book— ten pairs of arguments and counter-arguments supposedly representing his interlocutors' thoughts, elaborating them on the basis of specific statements such as "the divine power is the most powerful power". According to the disciple, such statements, since they are drawn from superlative degree predications of divine dignities,[7] would be "primary, true and necessary".[8] Finally, we should stress that for each of these arguments several consequences are put forward, whose further appraisals are extremely important. In general, all these arguments follow a uniform pattern which can be gathered from the following example:

> Divine Power is the most powerful power. Whatever is the most powerful power can exist and act most powerfully. God is the most powerful power; therefore, God can exist and act most powerfully. As a consequence, it follows that there is nothing that could ever force God not to be existing and acting most powerfully and there is nothing that with a finite power could escape from Him. On the contrary, it is argued

[6]"Duobus modis intendimus in parte ista argumentari, uidelicet faciendo duos syllogismos demonstratiuos, primitiuos, ueros et necessarios, et duos non ueros, ut per non ueros cognoscamus ueros, et e conuerso." [14, p. 87].

[7]Roughly, the divine dignities are constitutive principles of reality which are present in God.

[8]On the Lullian method of proof by superlative degree, see [4, pp. 245–250]

> that divine power is not most powerful. Whatever is not the most powerful power cannot exist and act most powerfully. Now God is not the most powerful power; therefore, God cannot exist and act most powerfully. As a consequence, it follows that there is something opposed to Him which has the same or even more power than He does, and which could hinder Him and escape from Him with a finite power or with an equal or higher power.[9]

Llull's disciple is convinced that the paired rebutting arguments which he opposes allow to determine the truth or falsehood of certain statements about given aspects of the divine nature, "because, if something is predicated about God, then it can be reduced to the aforementioned arguments".[10] In the second distinction, he conducts an exhaustive analysis of the theses as imputed to the Averroist, so as to refute them. Using such theses, he construes pairs of inconsistent statements, taking each group of antagonist rebutting arguments as a foundation to the respective statements of such inconsistent pairs. In what follows, the disciple presumes that the consequences which were previously linked with the rebutting arguments, within the referred antagonist groups, are entailed by the respective inconsistent statements. He eventually selects one of the statements while discarding the other, backing this on the proof of its consequences' falsehood, and on the fact that if the consequent of an entailment is false, its

[9] "Diuina potestas est potentissima. Quidquid est potestas potentissima, potest existere et agere potentissime. Deus est potestas potentissima; ergo Deus potest existere et agere potentissime. Ad consequentiam istius sequitur, quod non sit aliquid, quod possit cogere Deum, quin Deus sit existens et agens potentissime, et quod non sit aliquid, quod possit euadere ab ipso cum finito posse. In contrarium arguitur sic: Diuina potestas non est potentissima. Quidquid non est potestas potentissima, non potest existere et agere potentissime. Deus non est potestas potentissima; ergo Deus non potest existere et agere potentissime. Ad consequentiam istius sequitur, quod est aliquod ens sibi oppositum coaequatum aut magis potens, quod potest ipsum impedire, et ab ipso euadere cum finito posse, aut cum posse ei coaequato, aut cum posse altiori." [19, p. 174]

[10] "Ait Raimundus: Feci uiginti syllogismos oppositos, cum quibus potest homo de Deo cognoscere suum existere et agere. Et ideo, si de Deo aliquid praedicetur, ad ipsos syllogismos potest reduci, et cum ipsis cognosci, si propositio sit uera aut falsa, cum quidquid de Deo dici potest, sit in ipsis implicite aut explicite." [19, p. 177]

antecedent must also be false. The following appraisal of the thesis concerning the infinity of divine force is a suitable example of the aforementioned procedure:

> The ten previous syllogisms show that God is an infinite force; and if [He] were not an infinite force, there would be contradiction in the ten subsequent syllogisms. With the help of these very syllogisms, the human intellect acknowledges, supposing it is subtle, well grounded, judicious and non-perverted, that God is an infinite force. Yet, if he were to judge the opposite way, it would be perverted and non-judicious. This is self-evident. However, we shall try to expose this in some manner. If divine Power were not infinite, then there would be something preventing it from being infinite. Such hindrance, in turn, would be the best, because whatever exists in God is maximal. And it would be most eternal, for whatever exists in God is most eternal. And it would be most powerful, for whatever exists in God is most powerful. Thence there necessarily follows that the best is the worst; maximum is minimum and so on. But since such contradictions are impossible, therefore, God's being an infinite force is fully proved.[11]

Among the characteristic procedures of the contradictory-syllogisms-method, the final option for only one constituent within each pair of inconsistent statements is very neat; it shows that the occurrence of an inconsistency does not entail the trivialization

[11] "Quod Deus sit infiniti uigoris, decem praedicti syllogismi primi hoc testantur. Et si non est infiniti uigoris, decem syllogismi posteriores contradicunt. Intellectus autem humanus, discursus per ipsos syllogismos, posito quod sit subtilis, fundatus, discretus et non peruersus cognoscit, quod Deus est infiniti vigoris; et si iudicaret in opposito, esset peruersus et indiscretus. Et hoc per se patet. Tamen aliquo modo super hoc intendimus declarare. Si diuina potestas non est infinita, aliquid est, quod impedit, quod ipsa non sit infinita. Et illud impedimentum est optimum, cum quidquid sit in Deo, sit optimum. Et est maximum, cum quidquid sit in Deo, sit maximum. Et est aeternalissimum, cum quidquid sit in Deo, sit aeternalissimum. Et est potentissimum, cum quidquid sit in Deo, sit potentissimum. Ex quo sequitur necessarie, quod optimum sit pessimum, et maximum minimum etc. Et quia tales contradictiones sunt impossibiles, demonstratum est ergo, quod Deus est infiniti uigoris." [19, p. 178]

of the underlying logic, which in turn would be endowed with a paraconsistent status.[12] Indeed, an important quote from the *Liber de syllogismis contradictoriis*, where Llull's disciple points out to his interlocutor that both of them are actually not standing in contradiction, but rather in equivocation,[13] suggests (backed by the observation, in the *Liber contradictionis*, according to which, in a debate, rivals are not always exactly in contradiction)[14] that the components of the inconsistent statement pairs should not be interpreted as 'p' and 'not-p', but rather as 'possible-p' and 'possible not-p', respectively. In this case, it turns out that in

[12]Stated here in a sketchy fashion, if L is a logic construed as a structure $\langle F, \vdash \rangle$, enclosing both a set F of formulae built from the negation operator '$\neg$' and a consequence relation '$\vdash$' ranging over F; and if any subset Γ of F is a theory belonging to L, there follows that L includes, among its principles, not only (i) the principle of non-contradiction, according to which $\exists\Gamma\forall\varphi(\Gamma \nvdash \varphi \wedge \Gamma \nvdash \neg\varphi)$; (ii) the principle of non-triviality, according to which $\exists\Gamma\exists\psi(\Gamma \nvdash \psi)$; (iii) the principle of explosion or *ex contradictione quodlibet*, according to which $\forall\Gamma\forall\varphi\forall\psi(\Gamma, \varphi, \neg\varphi \vdash \phi)$, but also (iv) Γ is inconsistent if $\exists\varphi(\Gamma \vdash \varphi \wedge \Gamma \vdash \neg\varphi)$; (v) Γ is trivial if $\forall\psi(\Gamma \vdash \psi)$ and (vi) Γ is explosive if $\forall\varphi\forall\psi(\Gamma, \varphi, \neg\varphi \vdash \phi)$. A logic is said to be paraconsistent if it rejects the principle of explosion or simply if it involves an inconsistent, but non-trivial theory (see, for example, [13, pp. 287–393], [6, p. 89–109], and [7, pp. 791–912]). In a previous study, we have already depicted and evidenced paraconsistent features in Llull's theories (see [21, pp. 63–77]).

[13]"Auerroista, ait Raimundista, tu semper facis obiectiones cum tuo possibili, quod est inferius; et ego soluo tuas obiectiones cum possibili, quod est superius. Et ideo tu et ego sumus in aequiuocatione, non autem in contradictione." [18, p. 192]

[14]"Dixit Contradictio: Mea essentia est in anima considerata, contracta et concepta. Habeo duas species, unam intensam per impossibile, aliam extensam per possibile. Per impossibile, quia circa idem non sum ens reale, ut puta: Per impossibile est, quod illud, quod est, non sit, dum est; et quod fuit, non fuerit; et quod album est, dum est album, sit nigrum, et e conuerso; et huiusmodi. Alia autem species est per possibile; et oritur a prima per accidens. Quae causat entia contraria, sicut est contrarietas inter calidum et frigidum, inter uerum et falsum, et huiusmodi. Ecce qualis sum. Aliqui autem credunt disputare per meam qualitatem intensam, et disputant per extensam, et e conuerso; et sic quid mirum, si non possunt conuenire." [18, p. 139]. Indeed the exposed Lullian distinction between contradiction *per impossible* and contradiction *per accidens* seems to be analogous to Aristotle's distinction between contradiction and contrariety (see [10, p. 244]). Briefly said, Aristotle settles contradiction and contrariety as distinct, mainly because the latter allows intermediary terms while the former does not. Such intermediary terms, however, must belong to the same genus of things with respect to which they are intermediary terms, inasmuch as (i) they are items forcefully ranging over anything which is going to be turned into its contrary and (ii) there must be no other shifting between genus except for the one *per accidens* (*Met*. 1057a18-29).

his attempt to avoid deriving a contradiction from an inconsistency —since 'possible-p and possible not-p' does not follow from 'possible-p, possible not-p'— the approach takes on an adaptive standpoint in relation to the occurrence of inconsistencies. Thus it avoids driving the underlying logic into trivialization, without completely giving up the inferential force of classical logic. Therefore, it seems suitable to claim that after detecting an inconsistency, the underlying logic of the contradictory-syllogisms-method behaves in a paraconsistent manner in inconsistent contexts, nullifying the application of some classic inference rules, although it may also allow, in consistent contexts, an unrestricted application of such full-fledged rules.[15]

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[15] The conceptual resemblance between the underlying logic of Llull's method of contradictory syllogisms and the discursive logic D_2^r of adaptive inconsistency is particularly remarkable, since they do not only share the paraconsistent character and the ban on the derivation of contradictions, but also they are both entirely definable in terms of classical logic ([12]. For further reading on inconsistency-adaptive logics, from a general point of view, see [2]).

Bibliography

[1] Aristotle. *Metaphysics*. Oxford, Clarendon Press, 1924.

[2] D. Batens. *A Survey of Inconsistency-adaptive Logics*. Batens, D. et al. (eds), Frontiers of Paraconsistent Logic. Baldock, UK, Research Studies Press, 49–73, 2000.

[3] P. Besnard and A. Hunter. *Elements of Argumentation*. Cambridge, Mass., MIT Press, 2008.

[4] A. Bonner. *Syllogisms, Fallacies and Hypotheses: Llull's New Weapons to Combat the Parisian Averroists*. Domínguez, F. et al. (eds), Aristotelica et lulliana. Steenbrugge/The Hague, Abbatia Sancti Petri/MNI, Instrumenta patristica XXVI, 457–475, 1995.

[5] A. Bonner. *The Art and Logic of Ramon Llull. A User's Guide*. Leiden, Brill, 2007.

[6] W. Carnielli and J. Marcos. *Ex contradictione non sequitur quodlibet*. Bulletin of Advanced Reasoning and Knowledge, 1, 89–109, 2001.

[7] N. Da Costa et al. *Paraconsistent Logics and Paraconsistency*. Jacquette, D. et al. (eds), Philosophy of Logic. Amsterdam, North-Holland, 791–912, 2007.

[8] A. Fidora and E. Rubio (eds), *Raimundus Lullus. An Introduction to his Life, Works and Thought*. Turnhout, Brepols, 2008.

[9] R. Imbach. *Lulle face aux Averroïstes parisiens*. Raymond Lulle et le Pays d'Oc. Toulouse, Édouard Privat, Cahiers de Fanjeaux 22, 261–282, 1987.

[10] M. Johnston. *The Spiritual Logic of Ramon Llull*. Oxford, Clarendon Press, 1987.

[11] C. Lohr. *Ramon Llull*. Gracia, J., Noone, T. (eds), A Companion to Philosophy in the Middle Ages. Oxford, Blackwell, 2002.

[12] J. Meheus. *An Adaptive Logic Based on Jaśkowski's Approach to Paraconsistency*. Journal of Philosophical Logic, 35, 539–567, 2006.

[13] G. Priest. *Paraconsistent Logic*. Gabbay, D. et al. (eds), Handbook of Philosophical Logic. Dordrecht, Kluwer, vol. 6, 287–393, 2002.

[14] Ramon Llull. *Ars compendiosa Dei*. Ed. M. Bauzà (ROL XIII). Turnhout, Brepols, 1985, 15–331.

[15] Ramon Llull. *Excusatio Raimundi*. Ed. Ch. Lohr (ROL XI). Turnhout, Brepols, 329–375, 1983.

[16] Ramon Llull. *Liber, in quo declaratur, quod fides catholica est magis probabilis quam improbabilis*. Ed. H. Riedlinger (ROL VI). Turnhout, Brepols, 327–373, 1978.

[17] Ramon Llull. *Liber de refugio intellectus*. Ed. Ch. Lohr (ROL XI). Turnhout, Brepols, 223–329, 1983.

[18] Ramon Llull. *Liber contradictionis*. Ed. H. Harada (ROL VII). Turnhout, Brepols, 137–158, 1975.

[19] Ramon Llull. *Liber de syllogismis contradictoriis*. Ed. H. Harada (ROL VII). Turnhout, Brepols, 169–198, 1975.

[20] Ramon Llull. *Vita coaetanea*. Ed. H. Harada (ROL VIII). Turnhout, Brepols, 271–309, 1980.

[21] G. Wyllie. *La falacia luliana de la contradicción y la paraconsistencia*. Aquinate, 14, 63–77, 2011.

9 The Art of Definition: A Note on Ramon Llull and Charles S. Peirce

Alexander Fidora

It is a well know fact that the philosopher Charles Sanders Peirce (1839-1914), the founder of pragmatism and modern semiotics in the nineteenth century, was under a strong influence of medieval thought.

Peirce himself did not conceal his admiration for medieval philosophers, such as John Duns Scotus or William of Ockham, especially because of their rigorous application of logical analysis to the problem of universals, in other words, the ontological status of general terms.

Therefore, it is of no surprise that Peirce also knew the work of Ramon Llull. It is true that in some fragments of his oeuvre, the mentions of Llull are far from praise. In this manner, under the influence of the important, albeit tendentious history of Western logic by Carl Prantl, Peirce gets to talk about the "crazy Raymond Lully" treating him in one breath with the "stupid Albertus Magnus", the "superficial John of Salisbury" and the "insignificant Cusa" ([9], vol. I, p. 130). However, it is equally true that, at the same time, Llull is distinguished as "[one] of the most acute logicians" ([8], 3.465).

Be this as it may, there can be no doubt that Peirce was interested in the Majorcan's philosophical thought —an interest which can also be seen when one examines the list of Peirce's books. Thus, in addition to the books which Peirce could have consulted in the Harvard University Library (especially, Llull's *Arbor scientiae* which is preserved in three editions from the fifteenth and sixteenth century), we know that, as a part of his personal library, he was in possession of, at least, the important anthology of Llull's writings edited by Lazarus Zetzner.[1] Zetzner's *Raymundi Lulli Opera*, printed in four editions in Strasbourg during the sixteenth and seventeenth centuries, are one of the most influential works of European Lullism, since it was through these that many generations of erudite scholars came into contact with Llull and his *Ars*, such as, for instance, the young Leibniz. The *Raymundi Lulli Opera* consist of a series of Lullian works, as well as some Llull-related writings by Giordado Bruno, Agrippa of Nettesheim and others.[2]

One of the peculiarities of Llull's *Ars* is its very innovative use of logical devices, the most famous being the quasi-algebraic notation of its principles (each principle being represented by a letter, so that they can be easily combined). Among these innovative achievements stands Llull's curious theory of definition (cf. [2]). Thus, in his late *Ars brevis* (1308), contained in Zetzner's anthology, Llull gave the enigmatic definition of man as an *animal homificans* or an *animal cui proprie competit homificare*, i.e. an animal that 'manifies' or an animal whose proper characteristic is to 'manify'.

Llull was convinced that these definitions, which are based on the act proper (*actus proprius*) of the *definiens*, were much more precise and meaningful than the traditional Aristotelian definitions, which were based on the fixed properties of their object, e.g. the definition of man by genus and specific difference: *homo est animal rationale et mortale*. For Llull, the latter definitions would never allow their user to reach the very essence of a thing in the same way as his definitions do. This is why he defended his new approach energetically, and openly contrasted it with the classical theory of definition. In the *Ars brevis*, for instance, he writes ([5], vol. I, p. 628):

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[1]Cf. Peirce's ms. 179 "List of Books on Logic, Scholastic Philosophy, etc. from the Library of Prof. C. S. Peirce". It reads as follows: "*Raymundi Lullii Opera ea quae ad inventam ab ipso artem... pertinent*. With the *Clavis Artis lullianae* of J. H. Alsted, 2 vols. in 1. 8vo (1614)." (I owe this piece of information to José Vericat.) Apparently, the book consisted of two independent volumes which were re-bound together, namely, *Raymundi Lulli Opera* and *Clavis Artis lullianae* of Alsted, 1609.

[2]About Peirce and his reading of Zetzner's anthology see, in more detail, Fidora [1].

> Question: Whether a definition such as, Man is a manifying animal, or, Man is that being whose function is to manify, is more ostensive than the following one: Man is a rational, mortal animal. And one must reply that it is. The reason for this is that manification is something only proper to man, whereas rationality and mortality are proper to many things.

It is worth remarking that in this passage the criterion which Llull cites in favor of his definitions is their unequivocalness, for his dynamic definitions genuinely allow one to convert the *definiens* and the *definiendum*, without any of the logical problems which might arise from the fact that, in traditional definitions, specific differences constitute the species through their conjunction alone. Despite his enthusiastic defense of this "most easy and useful manner of defining", as Llull calls it in his *Ars generalis ultima* ([6], p. 23), he was well aware of its novelty as well as the criticisms to which it would be exposed by his Parisian colleagues, who, as he tells us, "despised [his] definitions with the teeth of dogs and the tongues of snakes" ([6], p. 23). Regardless of how much or how little Llull's contemporaries esteemed his seemingly trivial definitions, it seems that the idea was promised a future in modern logic. The following remarks are not intended to provide an exhaustive study of the development of modern theories of definition; they are designed to indicate, rather, certain interesting parallels between Llull and Peirce with regard to the latter's development of a theory of definition.

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As far as Peirce is concerned, scholars have observed a clear evolution of his doctrine of definition throughout his intellectual career. Peirce departs from a concept of definition which is not original at all, but, as he elaborates the program of philosophical pragmatism, he reviews and refines his doctrine of definition up to the point where he reaches a position of a high systematic interest (cf. [3], pp. 78-81).

It can be said that a conventional conception of the theory of definition predominates in his writings until the seventies of the nineteenth century. Thus, in his famous article "On a New List of Categories" ([11, pp. 49-59) from the year 1867, the American philosopher outlines a propositional logic in traditional terms of subject and predicate. Peirce gives an example of how this traditional approach is reflected in the theory of definition when he suggests the following definition of the mythological bird called griffin, half eagle, half lion. He says: "A griffin is a winged quadruped" (§4). This is of course the classic Aristotelian definition through genus —in this case, the four-legged animals—, and the specific difference, namely that this one is endowed with wings. Subsequently, Peirce proceeds to explain that the predicate, in other words, the specific difference, is a quality (§7), which, in turn, is the fixed result of a process of abstraction. Yet, since the seventies, and under the influence of the Logic of relations of Augustus De Morgan, Peirce radically revised his concept of definition —an undertaking which most probably occupied him for the rest of his life. The first indication of this change is found in another classic text, the review which Peirce dedicated in 1871 to the edition of the works of the eighteenth-century Irish bishop George Berkeley. This review is much more than a presentation of a book. In fact, Peirce used A. C. Fraser's edition as an occasion to systematically tackle one of the great problems of the history of philosophy, namely the discussion between nominalists and realists who debated for centuries on the ontological status of the general terms. Peirce examines this great metaphysical problem, which found an unusual solution in Berkeley, by placing it in a broader historical context, which recapitulates different stages of the history of philosophy. Therefore, a large part of his review is devoted to the medieval solutions to the problem of universals. It is precisely in this historical context that some reflections on the meaning of a proposition can be found, which are very relevant to our problem of the definitions. Specifically, Peirce introduces some key concepts such as the notion of "power" to capture the essence of things. The following statement gives a first impression of how Peirce uses this notion:

> To say that people sleep after taking opium because it has a soporific power, is that to say anything in the world but that people sleep after taking opium because they sleep after taking opium? ([8], 8.12)

At first glance, the statement is striking. What it seems to be saying is that the phrase "people sleep after taking opium because it has a soporific power" is trivial. But why? According to Peirce, this affirmation is trivial because it says nothing more than "people sleep after taking opium because they sleep after taking opium". And why does it not say anything else? The only possible answer is that the soporific power is part, analytically, of the very concept of opium, that is, of its definition! Peirce himself points in this direction when, subsequently, he adds the following remark:

> Is the present existence of a power in a thing anything in the world but a regularity in future events relating to a certain thing regarded as an element that is to be taken account of beforehand, in the conception of that thing? ([8], 8.12)

The answer to the earlier posed question is formulated here on an abstract plan. The phrase "people sleep after taking opium because it has a soporific power" is trivial because the soporific power of opium, which establishes a certain regularity in future events, namely, the fact that people usually sleep, is already contained, in advance, analytically within the very concept we have of opium, in other words, it is part of its definition.

The concept of definition mentioned in this second text, from 1871, is, therefore, clearly different from the classical theory of definition. The protagonists of this concept of definition are neither genus nor specific differences; rather than emphasising the fixed and static qualities, we are now faced with another, more dynamic, language that speaks of "power", "regularity" and "future events".

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A few years later, in 1877/78, Peirce published two articles which can be considered the founding essays of his pragmatism. The studies in question are "The Fixation of Belief" and "How to Make Our Ideas Clear".

In these, Peirce approaches the epistemological problem of how to give our opinions a solid basis, and he does it, firstly, from a historical perspective, as we have already seen in the review of Berkeley, namely, he reviews different stages of the history of epistemology starting from medieval times.

The central terms of this historical approach are the epistemological concepts of "doubt" and "belief". In fact, for Peirce doubt is the starting point of any process of knowledge; the reactions to doubt are different degrees of belief up to, ultimately, the definition of the doubtful terms.

This is the conceptual framework within which Peirce attempts to develop a systematic response to the question how to reach epistemologically justified beliefs. In "How to Make Our Ideas Clear" he explains, firstly, that our beliefs

> [are] distinguished by the different modes of action to which they give rise. If beliefs do not differ in this respect, if they appease the same doubt by producing the same rule of action, then no mere differences in the manner of consciousness of them can make them different beliefs. ([8], 5.398)

A belief or opinion that solves a doubt is therefore characterised not by how it presents itself to our mind. Rather, the most characteristic feature of any belief or opinion is its peculiar "mode" or "rule of action". The main terms which were identified earlier in the review of Berkeley reappear in this text in a more precise manner. Thus, the regularity of future events, which was discussed in the previous text, is now described as a veritable "mode" or a "rule of action" and the dynamic aspect is emphasised even more. From here, there is only a small step to the famous maxim of Peirce, which represents the essence of his so-called pragmatism; it states as follows:

> Consider what effects, that might conceivably have practical bearings, we conceive the object of our conception to have. Then our conception of these effects is the whole of our conception of the object. ([8], 5.402)

This is a radical statement: while in the review of Berkeley Peirce introduced a certain relation between the concept of definition and the dynamic notions which describe the behavior of a thing, in these later texts, this relation is strictly determined as a relation of identity: hence, the concept of an object is nothing else than the very knowledge of its “effects” and its “mode of action”. In order to sum up the last two quotations of Peirce, it can be concluded that the concept of an object or its meaning, and this is what definitions establish, would not depend at all on knowing its genus, its specific differences, etc., but is, strictly speaking, equivalent to the knowledge of the “effects” and the “mode of action” of the object in question, because it is the behavior of a thing that constitutes its essence.

These dynamic definitions which are based on the “mode of action” or “actus proprius”, as Llull calls it, are at the centre of Peirce’s entire philosophical system. In fact, in a draft written in 1902, more than 20 years after the publication of “The Fixation of Belief” and “How to Make Our Ideas Clear”, Peirce goes back to the problem of definition, insisting, once again, on its importance. Thus, in a little known fragment, he regrets not having treated the subject of definition in more detail and he announces his intention to take up his previous reflections in order to develop “the whole theory of definition and discuss its principal forms”. Unfortunately, Peirce did not manage to execute this project. Only some notes have been preserved within the above mentioned fragment and, given their importance, they will be quoted here in their integrity. Looking back, and referring himself specifically to “How to Make Our Ideas Clear”, Peirce says the following:

> In 1877 [in fact 1878] I published a paper on this subject in which I set forth a doctrine called ‘pragmatism’ which has since been talked of. But I know more about the clearness of ideas than I did a quarter of a century ago. I there described three grades of clearness: first, that which results from familiar use of the conception; second, that which results from logical analysis, and is expressed by a formal definition; and third, that which results from understanding the practical implication of the conception. I propose in this memoir to develop these three grades with fullness and not in the sketchy manner of a magazine article. I shall give the whole theory of definition and discuss its principal forms. I shall show, I hope quite convincingly, the great harm done by that definition by abstraction of

> which the Germans are so fond. For instance, to define coryza, you direct a person to think of a man with a bad cold. Now take away his pocket-handkerchief. Then take away his watch, knife, pocket-book, loose change, keys, shirt-buttons, boots, gloves, and hat. Then successively take away his clothes, body, and soul; and what you have left is a beautifully clear notion of coryza. I shall explain the doctrine of pragmatism more fully, and guard against extravagant applications. Finally, I shall develop a fourth, and higher, grade of clearness, resulting from an appreciation of the intellectual relations of the definitum. [10]

There can be no doubt that this fragment, no matter how brief it is, is immensely profound. In fact, Peirce clearly rejects here the traditional concepts of definition and admits the need to continue working on the concept on which he had elaborated beforehand. Therefore, he points out, not without irony, that a formal definition which operates through abstraction leads nowhere. On the contrary, the path to follow is the dynamic definition which is at the base of his pragmatism and which now appears as "appreciation of the intellectual relations of the definitum". It cannot be denied that the latter formula, namely, the definition as "appreciation of the intellectual relations of the definitum" is somewhat enigmatic. But it can be intuited that it is very close to what Ramon Llull had in mind and that he would have probably subscribed to this definition of definition. Certainly, this is pure speculation, since it is chronologically impossible. However, what seems quite possible, if not likely, is that the definition of definition by Llull had an effect on Peirce, either directly, through Lullian texts contained within the anthology of Zetzner, or indirectly, through Leibniz and others.

Briefly: "Definitio", as Llull says in his *Logica nova*, "[...] est per actum proprium et necessarium potentiae" ([7], p. 100). This is, in Llull's Catalan: "Difinició es per actu propri e necessari de potencia" ([4], p. 83), or, in Peirce's words: through the "mode of action" of a "power" . . .

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I would like to conclude these reflections by pointing to the driving force behind this quest of a definition of definition. Because for both, Llull as well as Peirce, the desire for clarity, to which the definition responds, has an important origin: doubt. Both scholars base their epistemology upon doubt as a central gnoseological attitude.[3]

Indeed, from an epistemological perspective, it seems obvious that this conjunction between doubt on the one hand, and clarity of definition on the other is an indispensable precondition for any critical dialogue also today. Yet, this may sound easier than it really is, as Peirce himself warns us saying:

> Many and many a philosopher seems to think that taking a piece of paper and writing down 'I doubt that' is doubting it, or that it is a thing that he can do in a minute as soon as he decides what he wants to doubt. Descartes convinced himself that the safest way was to 'begin' by doubting everything, and accordingly he tells us he straightway did so [...]. Well I guess not; for genuine doubt does not talk of *beginning* with doubting. The pragmatist knows that doubt is an art which has to be acquired with difficulty [...]. ([8], 6.498)

Resuming, therefore, a notion of Peirce's which, in turn, was central to Llull, I would like to conclude this note by saying that more than a purely theoretical reflection, what we need today, from the epistemological point of view, is this: an Art, as Llull and Peirce call it, of doubt and of defining the problems!

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[3] As far as Peirce is concerned, the importance of the concept of doubt in his two studies "The Fixation of Belief" and "How to Make Our Ideas Clear" has already been highlighted. Regarding Llull, it should be remembered that the entire *Ars* is founded on doubt, as it is stated in the first of its fundamental questions, namely: Utrum? —Whether something is (x) or not.

Bibliography

[1] Alexander Fidora. *Peirce's Account of the Categories and Ramon Llull*. Studia lulliana 47 (2007), 175–186.

[2] Alexander Fidora. *From 'Manifying' to 'Pegasizing': Ramon Llull's Theory of Definition Between Arabic and Modern Logic*. Revista Mirabilia 7 (2007), 72–86, http://www.revistamirabilia.com/Numeros/Num7/numero7_5.html.

[3] Klaus Oehler. *Charles Sanders Peirce*. München 1993.

[4] Ramon Llull. *Lògica nova*. Ed. Antoni Bonner, NEORL II, Palma de Mallorca 1998.

[5] Ramon Llull. *Selected Works of Ramon Llull*. Transl. Anthony Bonner, 2 vols., Princeton (NJ) 1985.

[6] Raimundus Lullus. *Ars generalis ultima*. Ed. Alois Madre, ROL XIV, Turnhout 1986.

[7] Raimundus Lullus. *Logica nova*. Ed. Walter Euler, ROL XXIII, Turnhout 1998.

[8] Charles S. Peirce. *Collected Papers*. Ed. Charles Hartshorne, Paul Weiss and Arthur Burks, 8 vols., Cambridge (MA) 1931–1958.

[9] Charles S. Peirce. *Contributions to 'The Nation'*. Ed. Kenneth Laine Ketner and James Edward Cook, 4 vols., Lubbock 1975–1979.

[10] Charles S. Peirce. *Draft D — MS L75.287–288*. Ed. Joseph Ransdell,

http://www.cspeirce.com/menu/library/bycsp/l75/ver1/l75v1-09.htm.

[11] Charles S. Peirce. *Writings of Charles S. Peirce — vol. II*. Ed. Edward C. Moore, Bloomington (IN) 1984.

Llull's Life: A Chronology

1229	James I reconquers Mallorca from Muslim domination.
1232	Birth of Ramon Llull in Palma de Mallorca.
1257	Llull marries Blanca Picany, and enters the services of Prince James, son of James the Conqueror.
1263	Llull's "conversion to penitence" at the age of thirty: Llull decides to write the best book in the world against the errors of the unbelivers, to found monasteries in which the different languages necessary for mission might be taught and to dedicate his life to the service of Christ.
1265	After a pilgrimage to the Marian shrine at Rocamadour, in southern France, and to Santiago de Compostela, Llull meets with Ramon de Penyafort in Barcelona, who advises him to return to Majorca and there, rather than in Paris, to devote himself to study and contemplation. With a Muslim slave Llull begins nine years of linguistic and intellectual training.
1271–1274	At the end of his nine years of study, Llull writes his first works, the *Lògica d'Algatzell* — a paraphrase of al-Ghazali's *Maqasid al-falasifa*, and the monumental *Book of Contemplation*.
1274	Death of the slave who had taught Llull Arabic. Illumination on Mount Randa, first version of the Art, Llull's philosophico-theological system.
1276	A Papal Bull confirms the foundation of the Monastery of Miramar, on Majorca, financed by James II, in which thirteen Franciscans study oriental languages and the Art.
1283	In Montpellier, Llull writes the novel *Blaquerna* and elaborates upon his system in the *Ars demonstrativa*.
1287	First visit to the Papal Court.
1287–1289	First visit to Paris.

1292	In Rome, Llull (already 60 years old) writes his first work on the crusades.
1293	So-called "psychological crisis" in Genoa followed by Llull's first journey to North Africa.
1297–1299	Second stay in Paris.
1299	James II of Aragon gives persmission to Llull to preach in all the synagogues and mosques within his domains.
1301–1302	Journey to Cyprus, Lesser Armenia and possibly Jerusalem.
1303	Llull writes his *Logica nova* in Genoa.
1305	Llull begins the definitive formulation of his system, the *Ars generalis ultima*.
1307	Second trip to North Africa (Bejaia), where Llull is imprisoned for six months and, finally, expelled. Shipwrecked near Pisa.
1308	In Pisa Llull writes the *Ars brevis*, finishes the *Ars generalis ultima*, and re-writes the work begun in Bejaia and lost in the shipwreck, the *Disputatio Raimundi christiani et Homeri saraceni*.
1309–1311	Fourth and final stay in Paris, where Llull writes some thirty works, most of them directed against the Latin Averroists. In 1310, forty Masters and Bachelors of the University of Paris sign a document approving the *Ars brevis*. From Paris he travels to Vienne, recommeding to the Council of Vienne the foundation of language schools.
1313–1314	Visit to Sicily.
1314–1315	Third mission to North Africa (Tunis), where Llull dedicates works to the Sultan. His final works date from December 1315.
1316	Around or before March he dies on board the ship taking him back from Tunis to Majorca or in Majorca itself. Llull must have been 84 years old. His remains are burried in the Franciscan Monastery in Palma.

Recommendations for Further Reading

Llull's Works in English:

Anthony Bonner (ed. and trans.). *Selected Works of Ramon Llull*, 2 vols. Princeton 1985.

Anthony and Eve Bonner (ed. and trans.). *Doctor Illuminatus. A Ramon Llull Reader*. Princeton 1993.

http://lullianarts.net — Web page by Yanis Dambergs devoted to aspects of the mechanisms from the *Ars lulliana*, including translations of several of Llull's works.

Introductions to his Life and Thought in English:

Anthony Bonner. *The Art and Logic of Ramon Llull. A User's Guide*. Leiden/Boston 2007.

Alexander Fidora and Josep Enric Rubio (eds.). *Raimundus Lullus. An Introduction to his Life, Works and Thought*. Turnhout 2008.

Martin Gardner. *Logic Machines and Diagrams*. Chicago 1982.

Jocelyn N. Hillgarth. *Ramon Lull and Lullism in Fourteenth-Century France*. Oxford 1971.

Updated General Bibliography:

For a continuously updated database regarding Ramon Llull's bibliography, cf. the web page belonging to the Centre de Documentació Ramon Llull of the University of Barcelona: http://orbita.bib.ub.es/llull/

Digitalised manuscripts:

From the following web page, maintained by the Raimundus-Lullus-Institut of the University of Freiburg i. Br., one can gain access to digitised images of the manuscripts of Ramon Llull's works as held in libraries from all around the world: http://freimore.uni-freiburg.de/lullus/index.html

Index