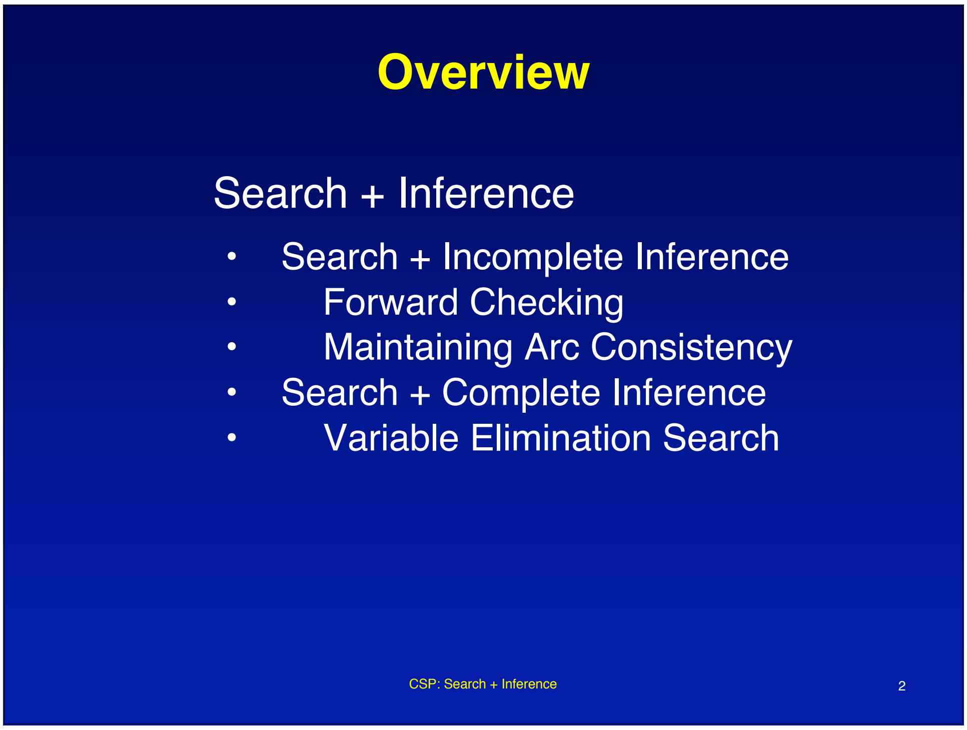
CSP: Search + Inference

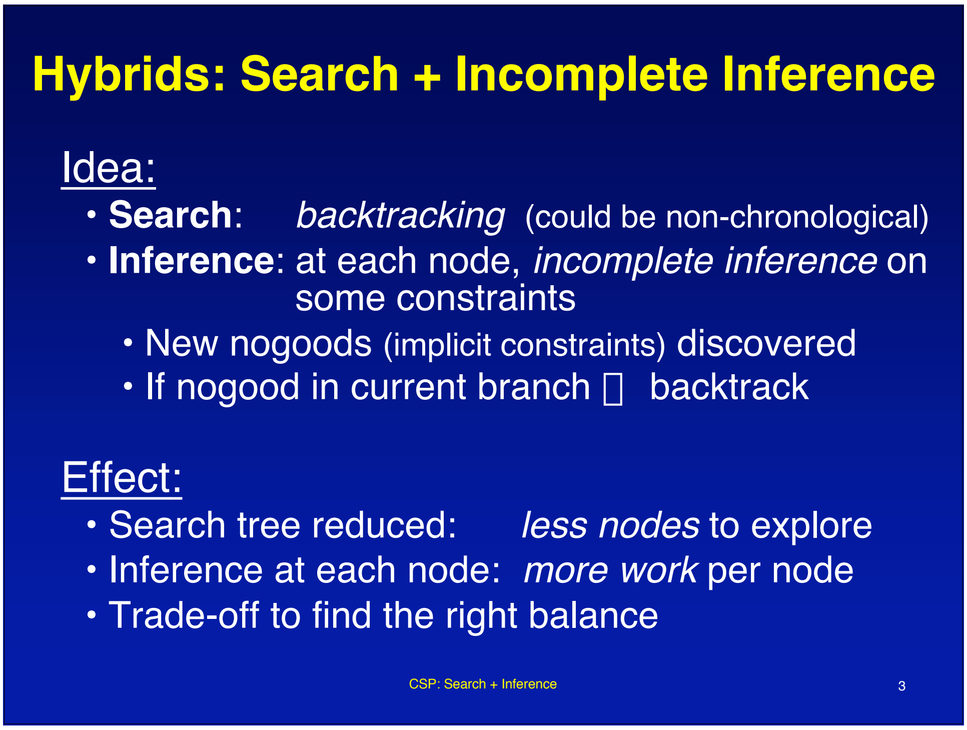
Pedro Meseguer
IIIA-CSIC
Bellaterra, Spain



Overview

Search + Inference

- Search + Incomplete Inference
    - Forward Checking
    - Maintaining Arc Consistency
- Search + Complete Inference
    - Variable Elimination Search



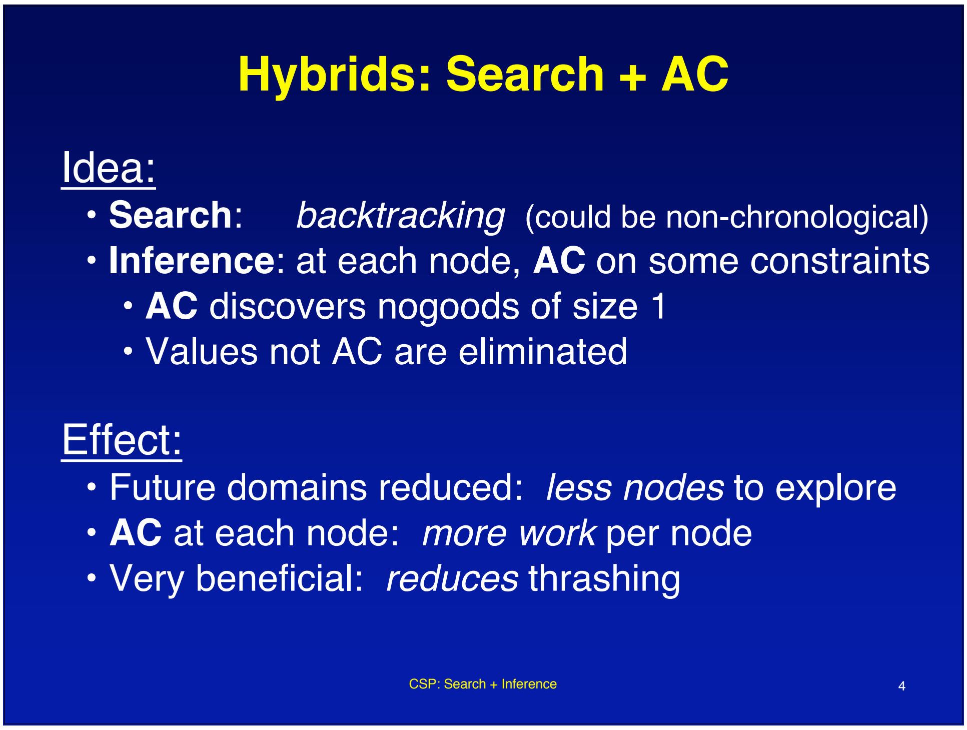
Hybrids: Search + Incomplete Inference

Idea:

- **Search**: *backtracking* (could be non-chronological)
- **Inference**: at each node, *incomplete inference* on some constraints
  - New nogoods (implicit constraints) discovered
  - If nogood in current branch ⇒ backtrack

Effect:

- Search tree reduced: *less nodes* to explore
- Inference at each node: *more work* per node
- Trade-off to find the right balance



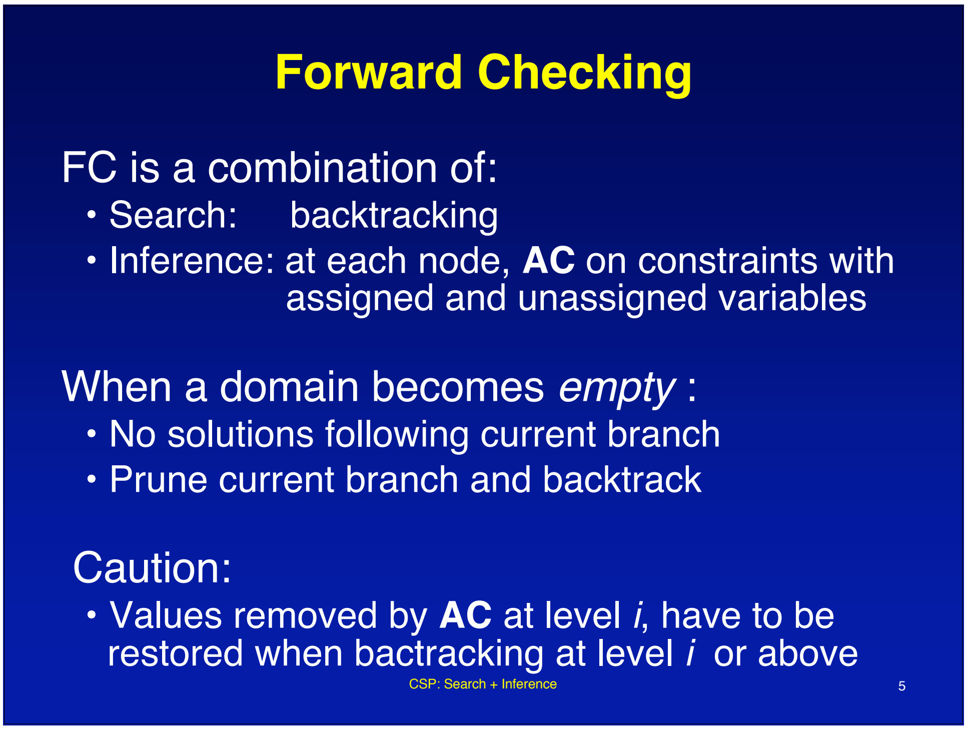
Hybrids: Search + AC

Idea:

- **Search**: *backtracking* (could be non-chronological)
- **Inference**: at each node, **AC** on some constraints
  - **AC** discovers nogoods of size 1
  - Values not AC are eliminated

Effect:

- Future domains reduced: *less nodes* to explore
- **AC** at each node: *more work* per node
- Very beneficial: *reduces* thrashing



Forward Checking

FC is a combination of:

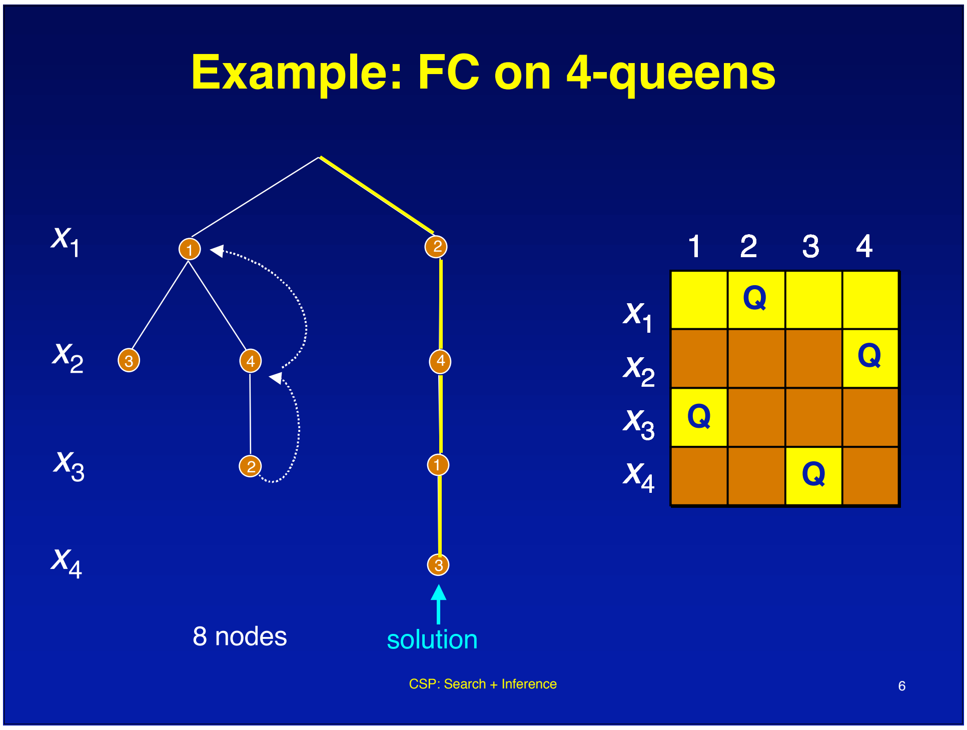
- Search: backtracking
- Inference: at each node, **AC** on constraints with assigned and unassigned variables

When a domain becomes *empty* :

- No solutions following current branch
- Prune current branch and backtrack

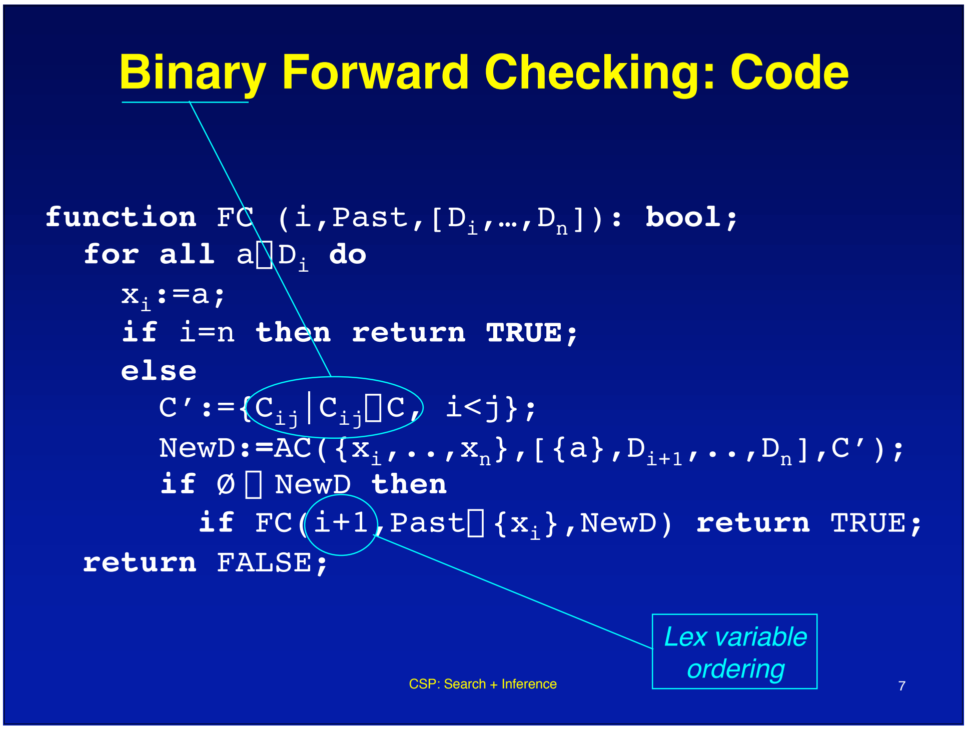
Caution:

- Values removed by **AC** at level *i*, have to be restored when bactracking at level *i* or above



Example: FC on 4-queens

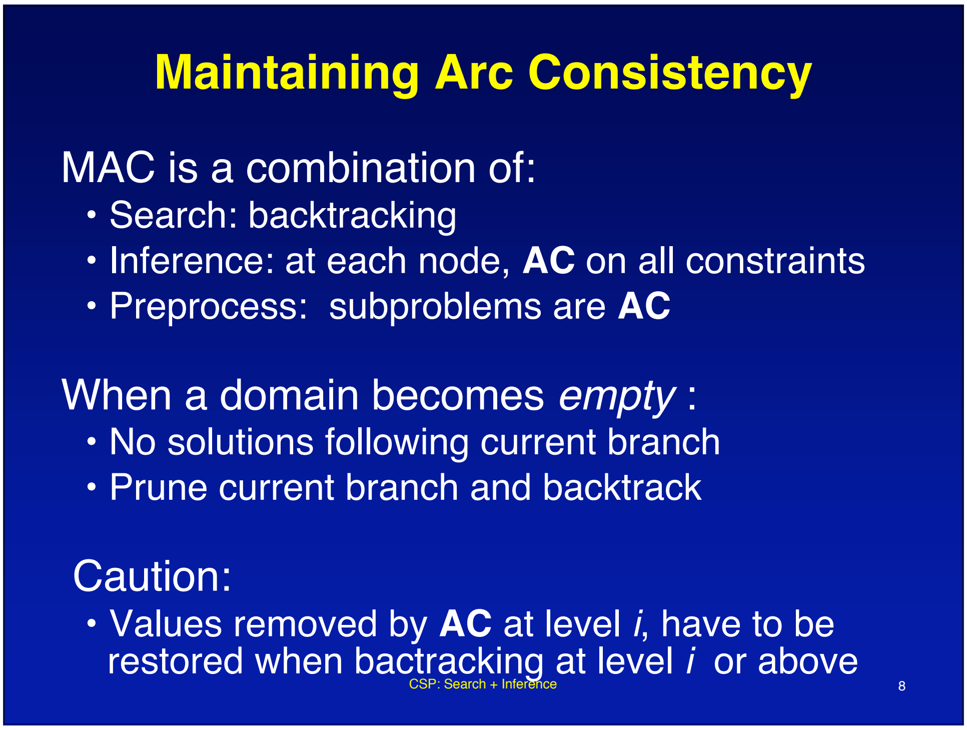
	1	2	3	4
x_1		Q		
x_2				Q
x_3	Q			
x_4			Q	



Binary Forward Checking: Code

```
function FC (i,Past,[$D_i$,…,$D_n$]): bool;
  for all a□$D_i$ do
    $x_i$:=a;
    if i=n then return TRUE;
    else
      C':={$C_{ij}$|$C_{ij}$□C, i<j};
      NewD:=AC({$x_i$,..,$x_n$},[{a},$D_{i+1}$,..,$D_n$],C');
      if ∅□NewD then
        if FC(i+1,Past□{$x_i$},NewD) return TRUE;
  return FALSE;
```

Lex variable ordering



Maintaining Arc Consistency

MAC is a combination of:

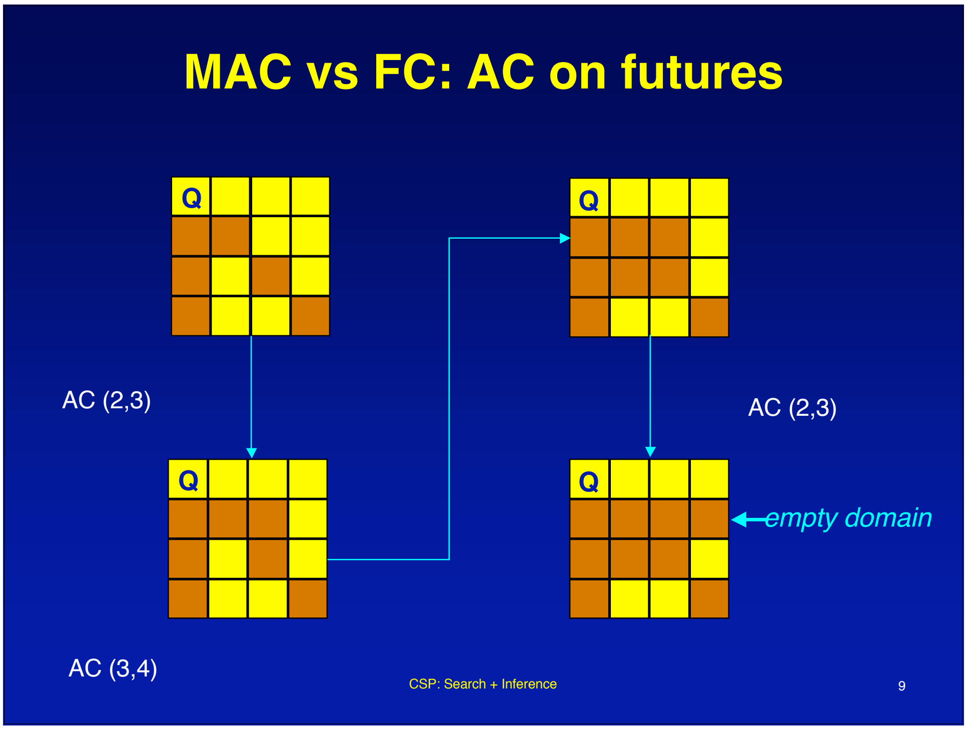
- Search: backtracking
- Inference: at each node, **AC** on all constraints
- Preprocess: subproblems are **AC**

When a domain becomes *empty* :

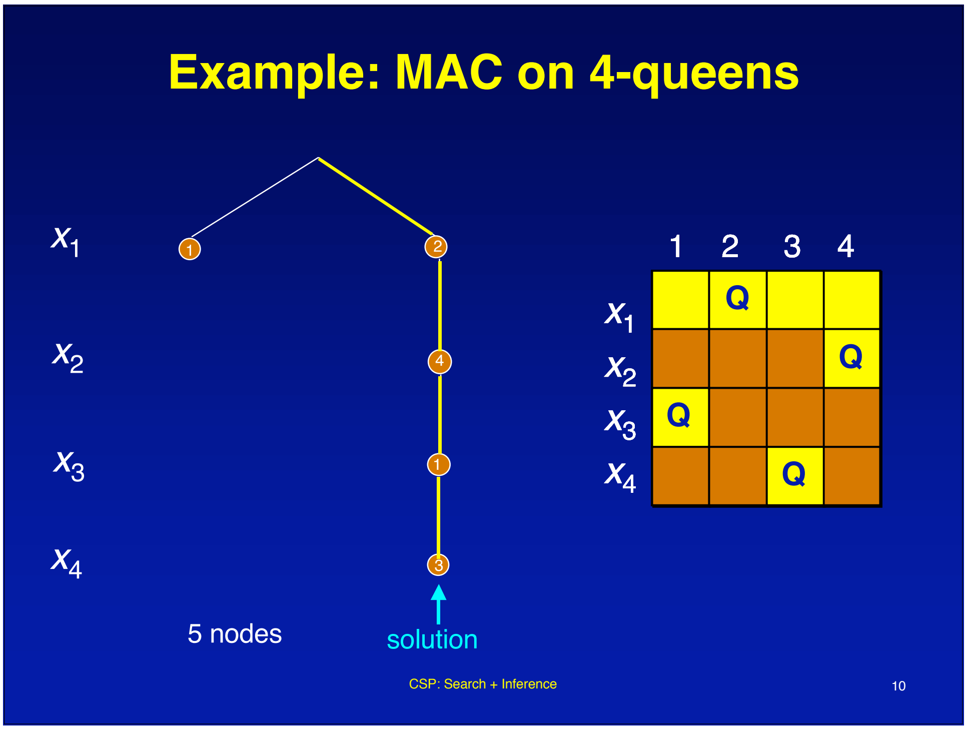
- No solutions following current branch
- Prune current branch and backtrack

Caution:

- Values removed by **AC** at level *i*, have to be restored when bactracking at level *i* or above



MAC vs FC: AC on futures

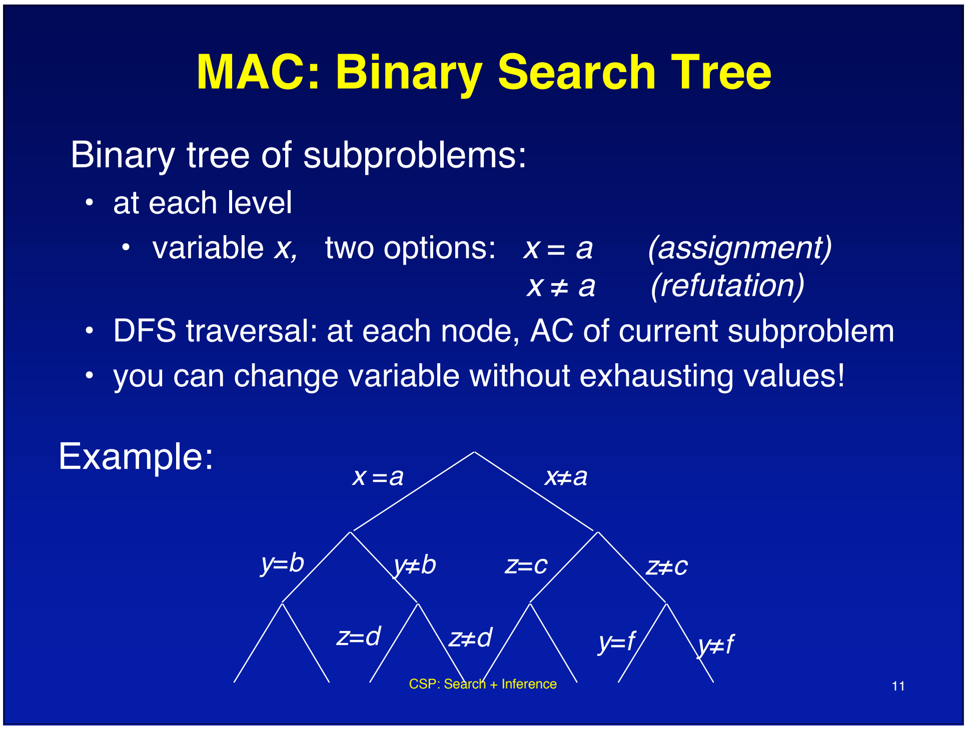


Example: MAC on 4-queens

5 nodes

solution

	1	2	3	4
x_1		Q		
x_2				Q
x_3	Q			
x_4			Q	

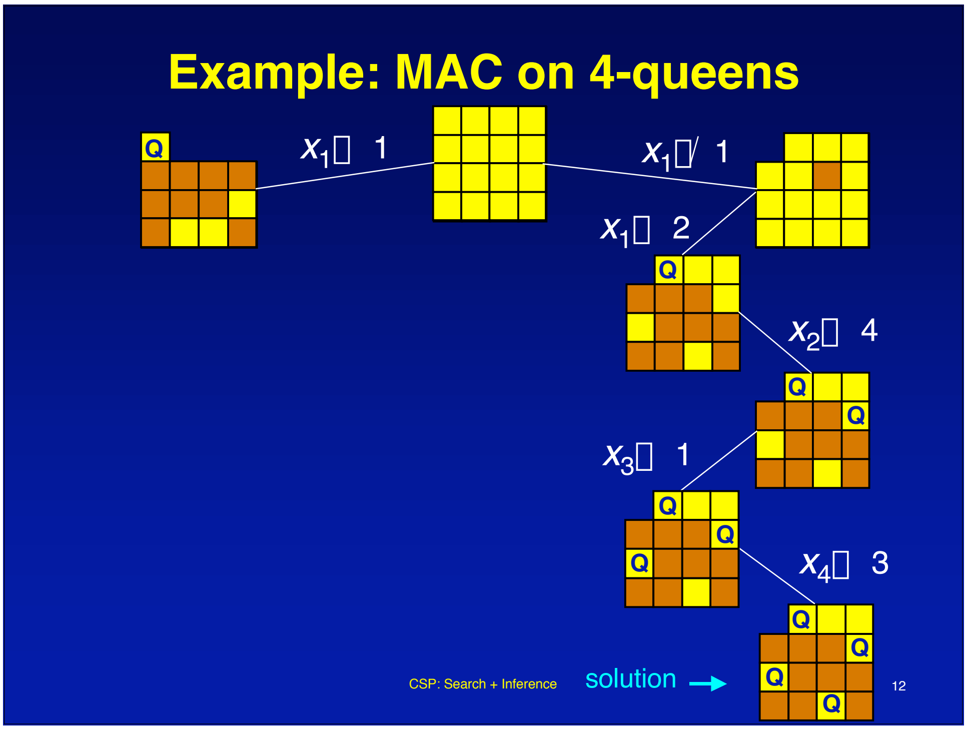


MAC: Binary Search Tree

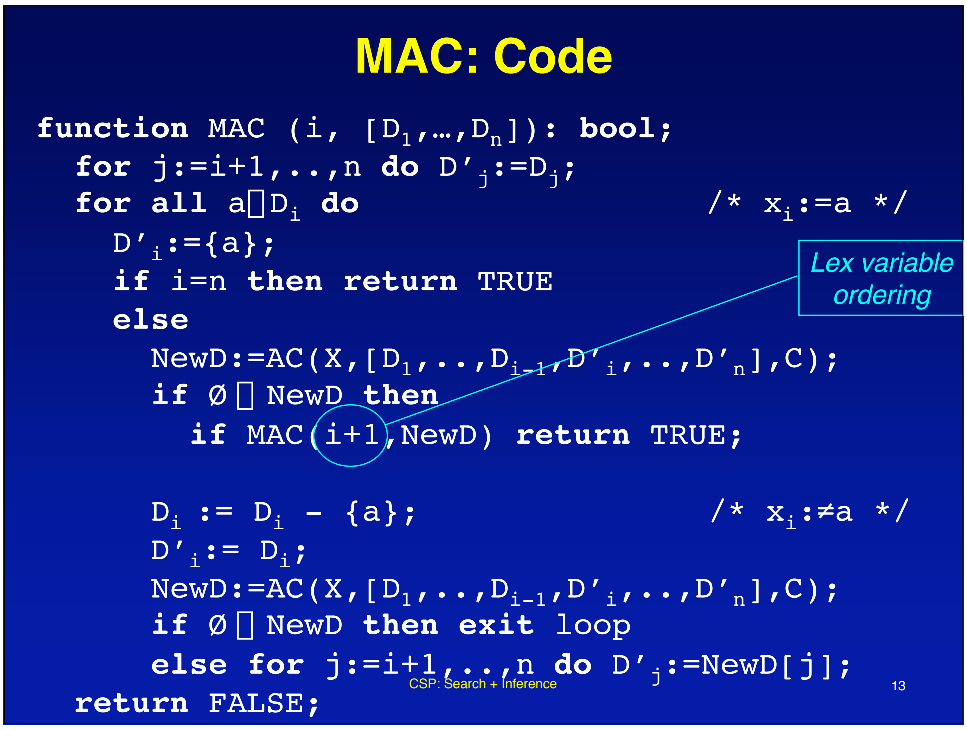
Binary tree of subproblems:

- at each level
  - variable *x*, two options: *x = a* *(assignment)*
x ≠ a *(refutation)*
- DFS traversal: at each node, AC of current subproblem
- you can change variable without exhausting values!

Example:



Example: MAC on 4-queens

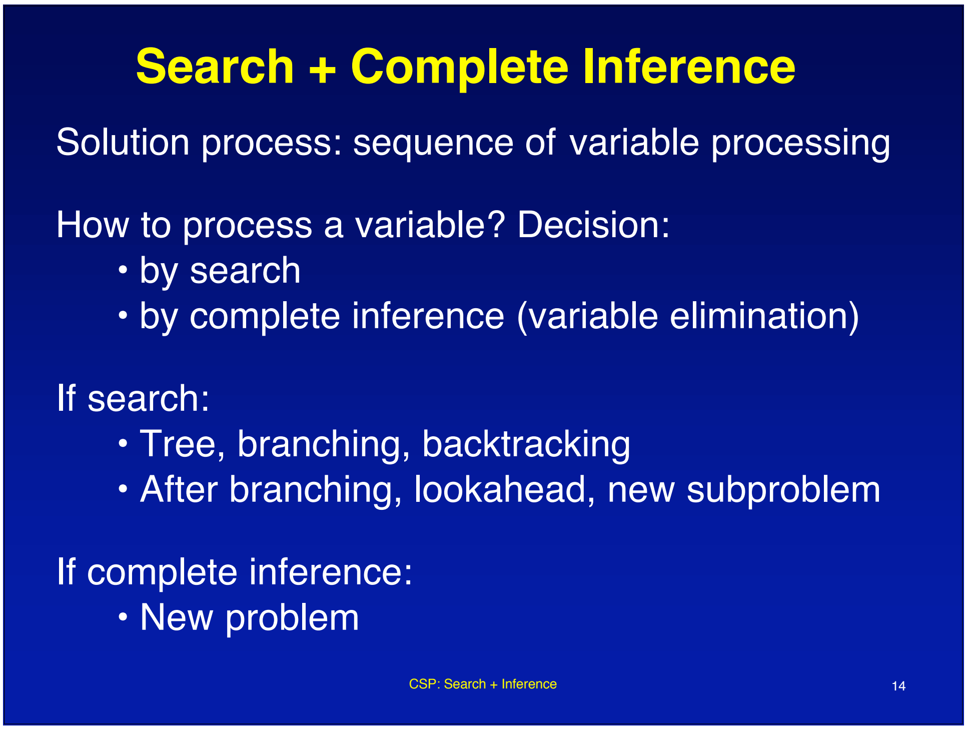


MAC: Code

```
function MAC (i, [$D_1$,…,$D_n$]): bool;
  for j:=i+1,..,n do $D'_j$:=$D_j$;
  for all a□$D_i$ do                    /* $x_i$:=a */
    $D'_i$:={a};
    if i=n then return TRUE
    else
      NewD:=AC(X,[$D_1$,..,$D_{i-1}$,$D'_i$,..,$D'_n$],C);
      if Ø□NewD then
        if MAC(i+1,NewD) return TRUE;

      $D_i$ := $D_i$ - {a};                  /* $x_i$:≠a */
      $D'_i$:= $D_i$;
      NewD:=AC(X,[$D_1$,..,$D_{i-1}$,$D'_i$,..,$D'_n$],C);
      if Ø□NewD then exit loop
      else for j:=i+1,..,n do $D'_j$:=NewD[j];
  return FALSE;
```

Lex variable ordering



Search + Complete Inference

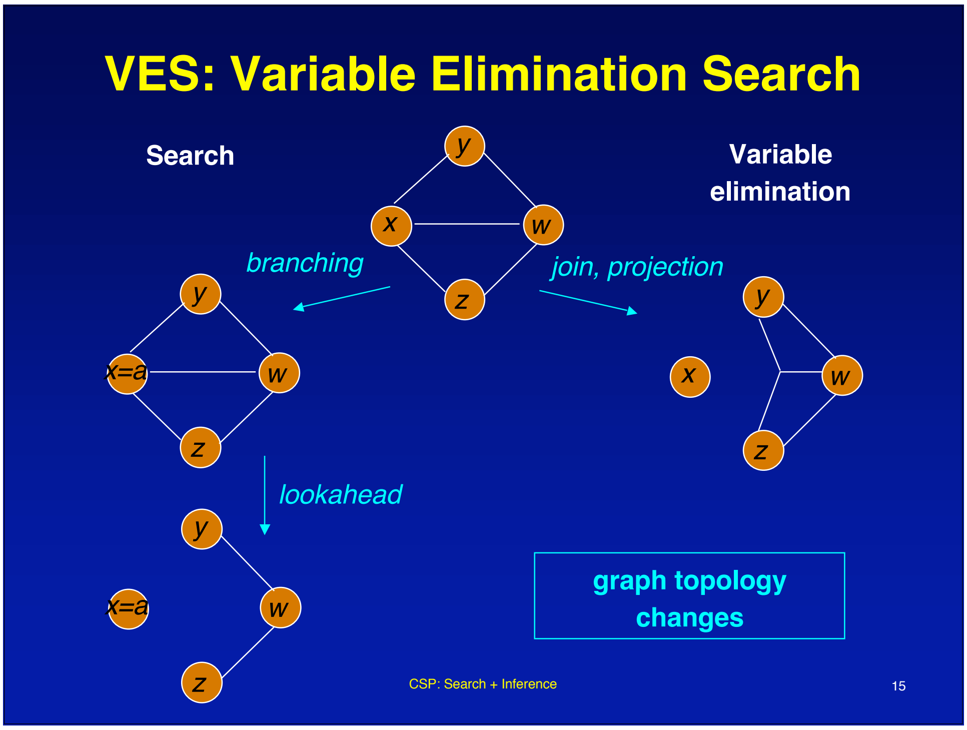
Solution process: sequence of variable processing

How to process a variable? Decision:
- by search
- by complete inference (variable elimination)

If search:
- Tree, branching, backtracking
- After branching, lookahead, new subproblem

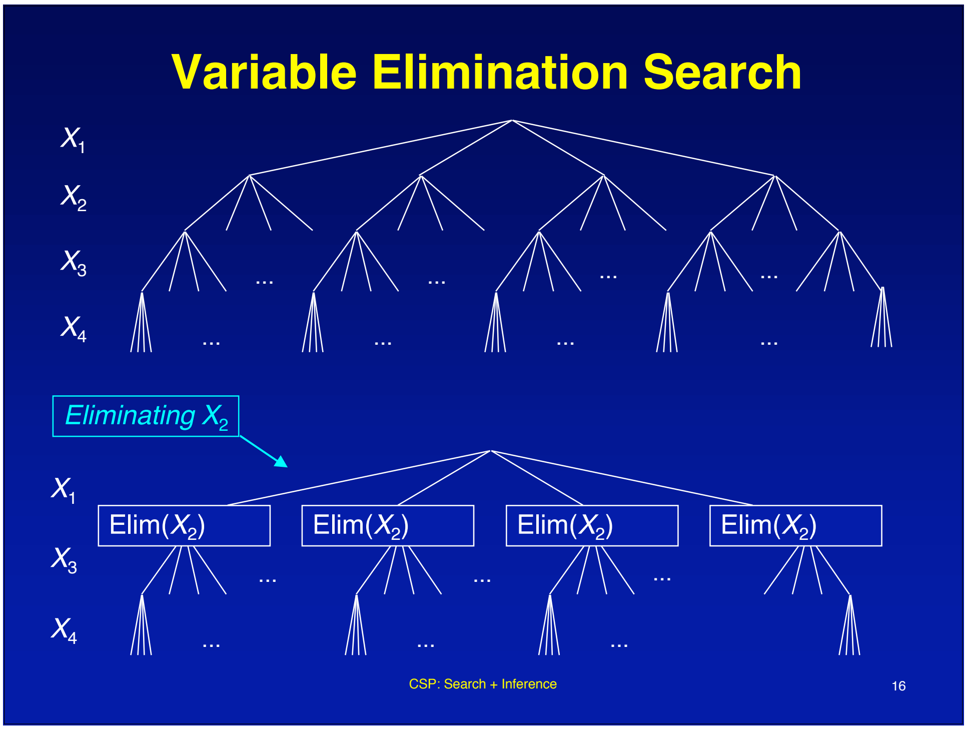
If complete inference:
- New problem

VES: Variable Elimination Search



graph topology changes

Variable Elimination Search



VES (k)

- At each node:

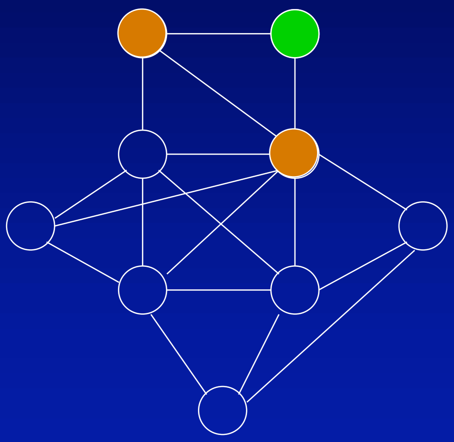
```
$x_i$ □ select a future variable
**if** *degree*($x_i$) □ $k$ **then** eliminate $x_i$
**else** branch on the values of $x_i$
        perform lookahead on branching
```

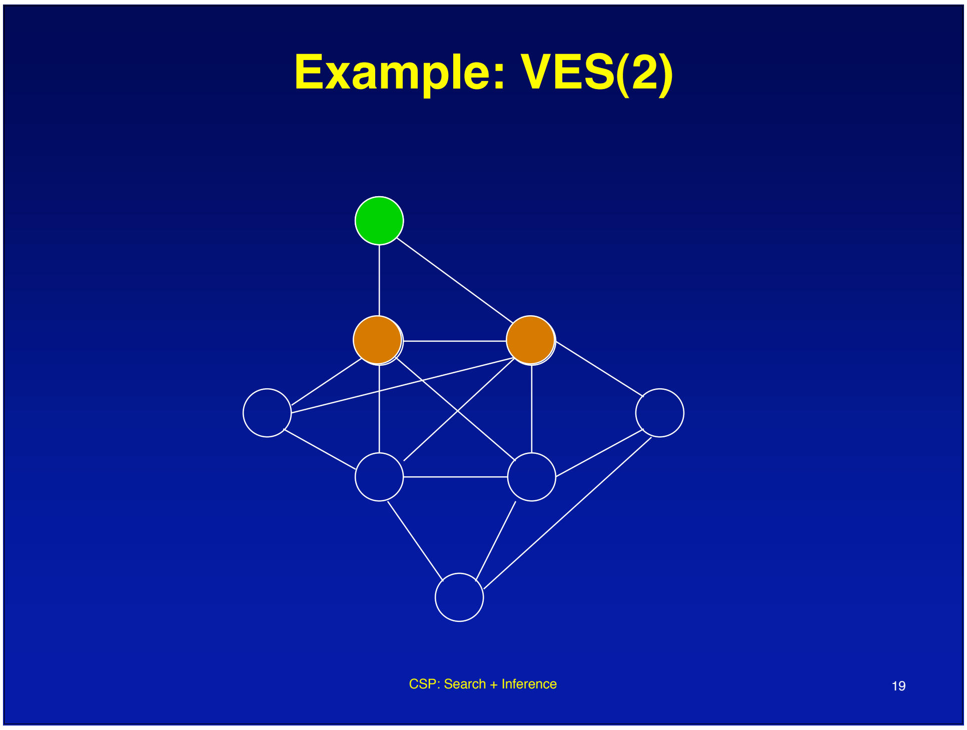
- Property:

  VES(-1) is search

  VES(w*) is complete inference

Example: VES(2)





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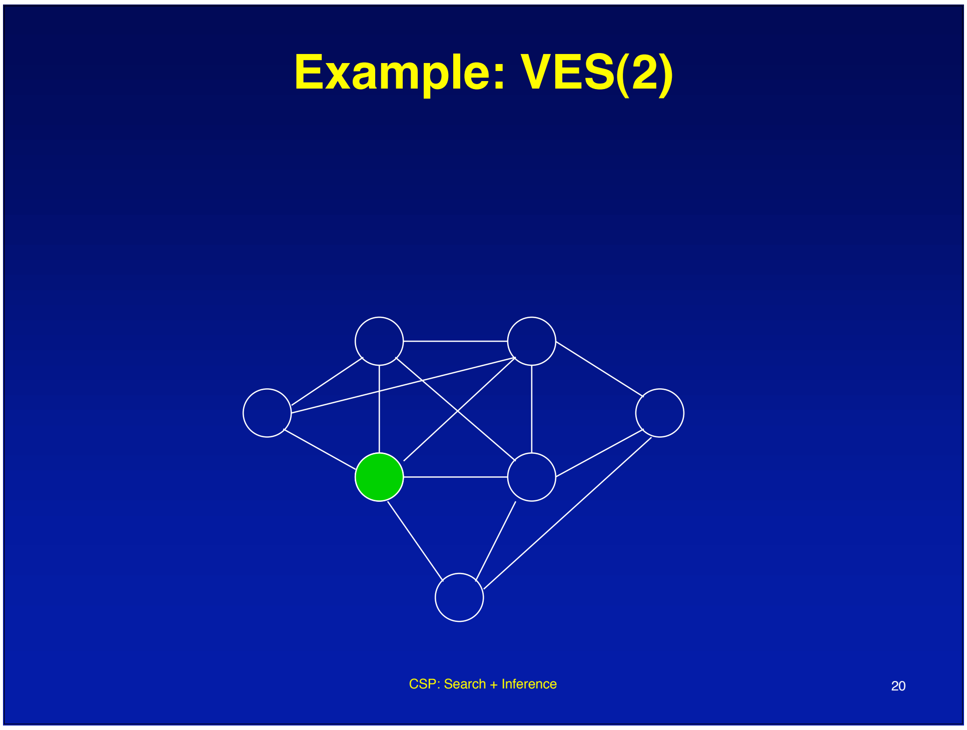
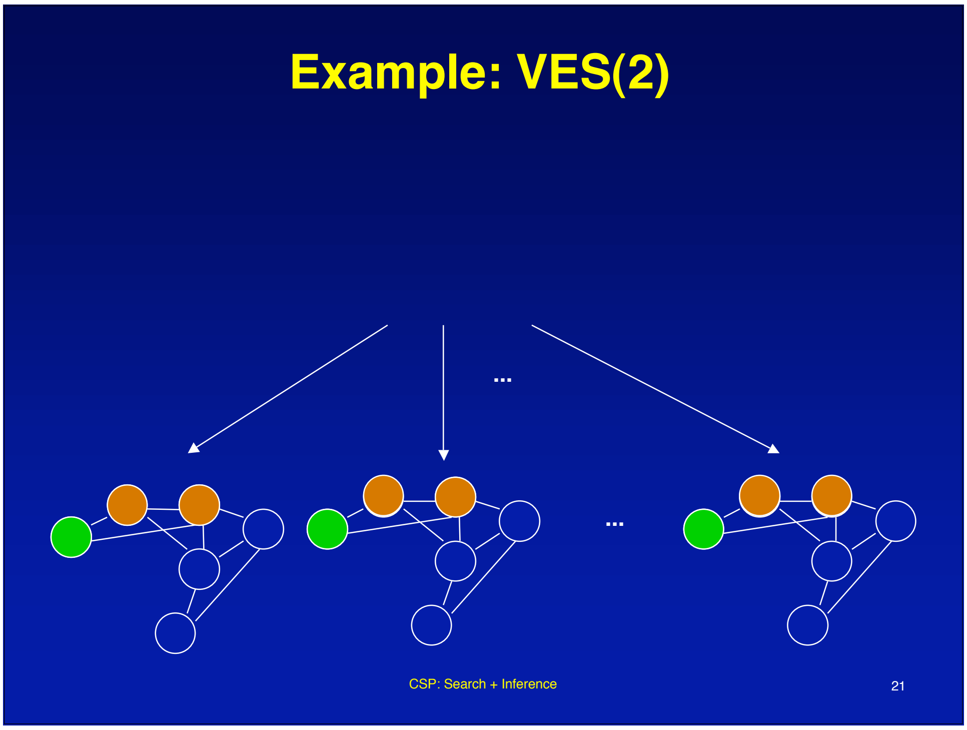


Example: VES(2)



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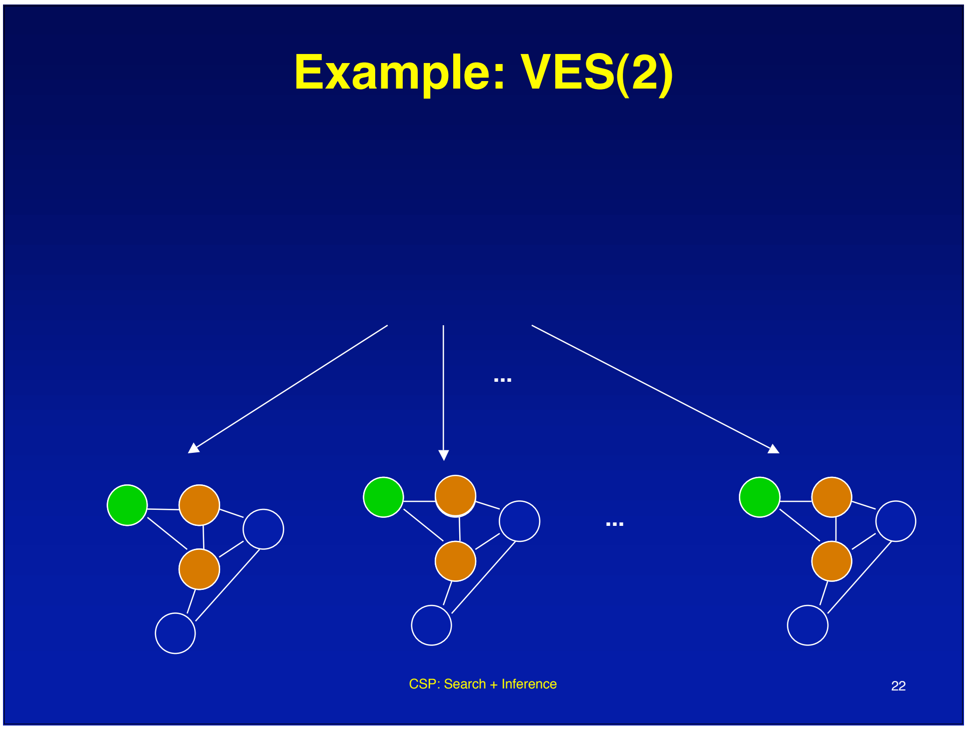

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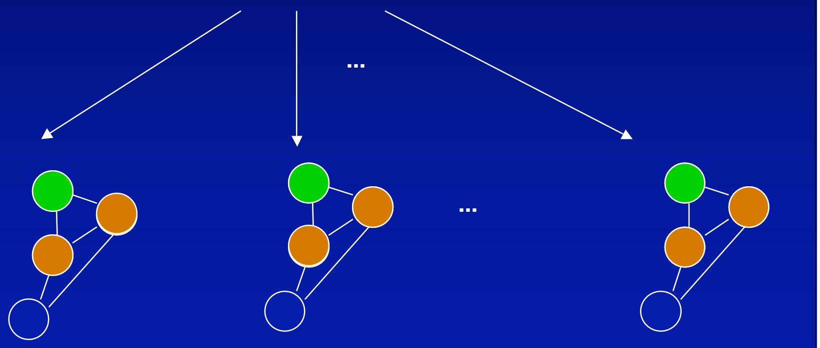

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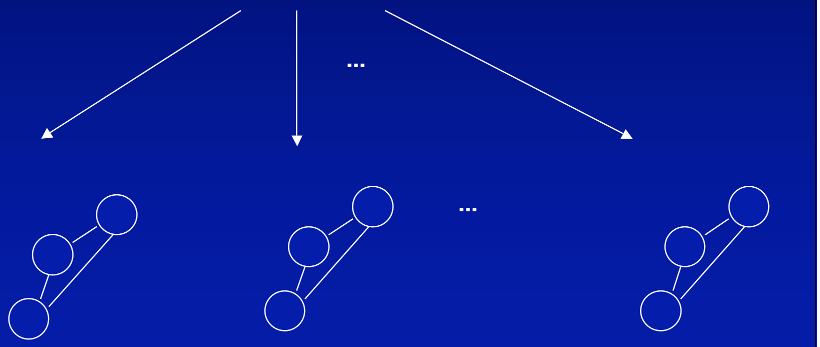

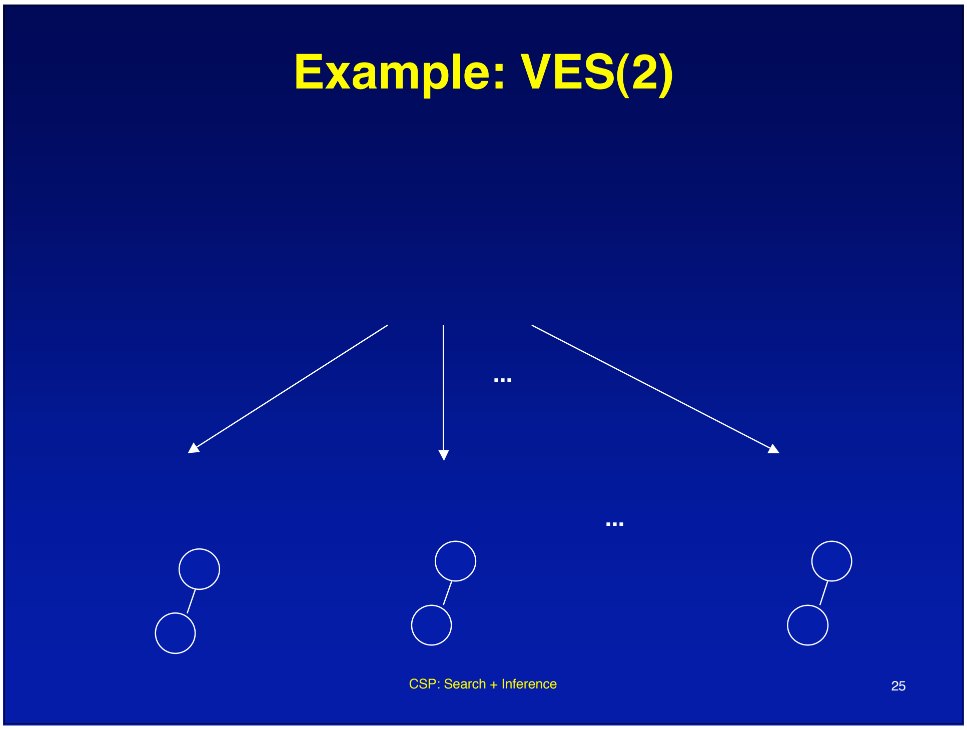

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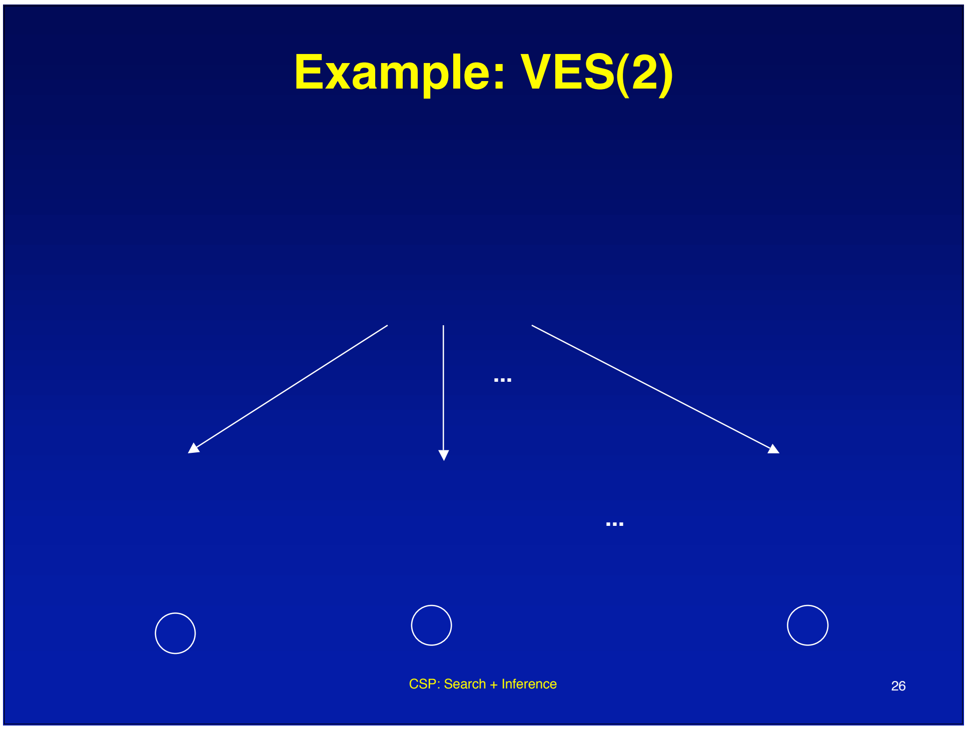

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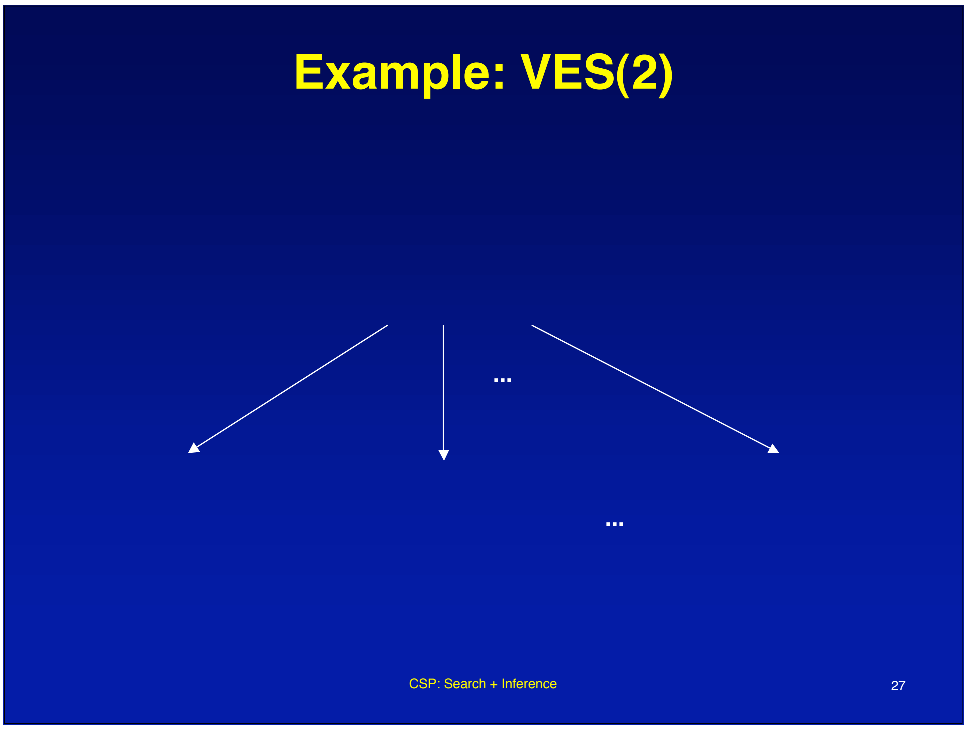

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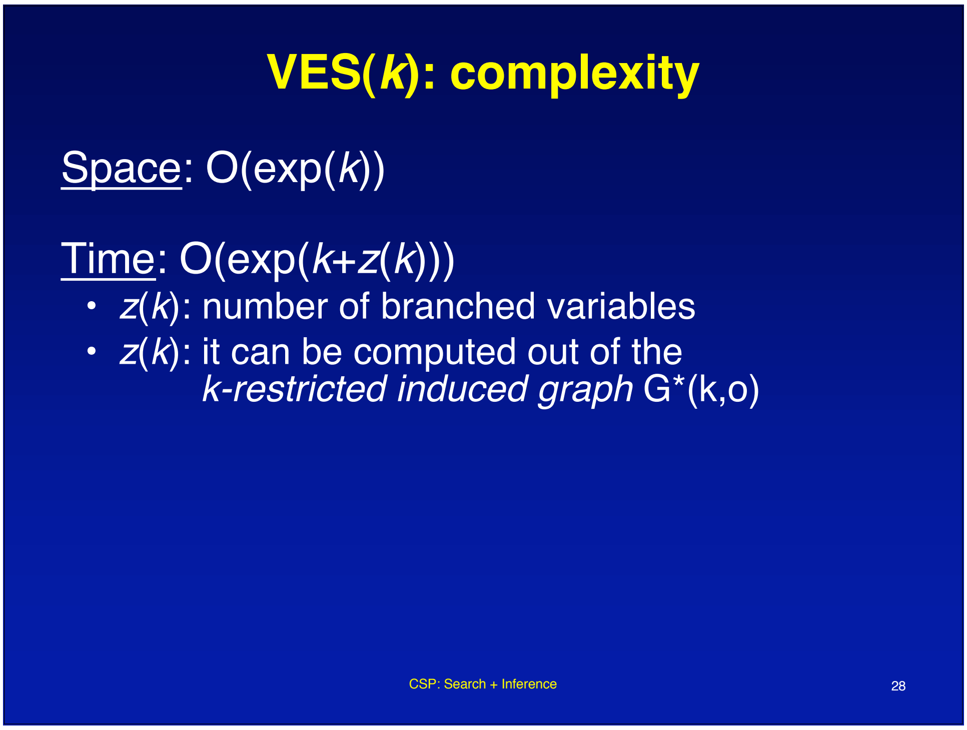

VES(k): complexity

Space: O(exp(k))

Time: O(exp(k+z(k)))

- z(k): number of branched variables
- z(k): it can be computed out of the *k-restricted induced graph* G*(k,o)