VIENNA SUMMER OF LOGIC 2014

Abstract Booklet

Logic Colloquium

Logic, Algebra and Truth Degrees

Vienna Summer of Logic, July 9–24, 2014

organized by the kurt gödel society

[3] M. Baaz, A. Ciabattoni and C. Fermüller, Cut-Elimination in a Sequents-of-Relations Calculus for Gödel Logic. In Proceedings of ISMVL 2001, pp. 181-186. IEEE.

[4] M. Baaz and C. Fermüller, Analytic Calculi for Projective Logics. Proceedings of Automated Reasoning with Tableaux and Related Methods (Tableaux'99). LNAI 1617, pp. 36-51. 1999.

[5] M. Baaz, O. Lahav and A. Zamansky. Finite-valued Semantics for Canonical Labelled Calculi. *J. Autom. Reasoning* 51(4): 401-430, 2013.

[6] M. Bongini, A. Ciabattoni, F. Montagna, Proof Search and Co-NP Completeness for Many-Valued Logics, Preprint 2013, submitted.

[7] S. Bova, F. Montagna. Proof Search in Hajek's Basic Logic, *ACM Transactions on Computational Logic*, 9: 1-26, 2008.

[8] A. Ciabattoni, C. Fermüller and G. Metcalfe. Uniform Rules and Dialogue Games for Fuzzy Logics. Proceedings of LPAR 2004. LNAI 3452, pp. 496-510. 2004.

[9] A. Ciabattoni, F. Montagna. Proof Theory for Locally Finite Many-Valued Logics: Semi-projective Logics. *Theoretical Computer Science* 480: 26-42, 2013.

[10] P. Hájek, "Metamathematics of Fuzzy Logic", Kluwer Acad. Publ., Dordrecht 1998.

[11] R. Horčík, K. Terui. Disjunction Property and Complexity of Substructural Logics. *Theoretical Computer Science* 412(31): 3992-4006, 2011.

[12] G. Metcalfe, N. Olivetti and D. Gabbay, "Proof Theory for Fuzzy Logics", Springer Series in Applied Logic Vol. 36, 2008.

[13] T. Vetterlein. Analytic Calculi for Logics of Ordinal Multiples of Standard t-Norms, *J. of Logic and Computation*, 18(1): 35-57, 2008.

▸ FÉLIX BOU, *Introducing an exotic MTL-chain.*
Artificial Intelligence Research Institute, IIIA-CSIC.
E-mail: fbou@iiia.csic.es.

Acknowledgment. The project COINVENT acknowledges the financial support of the Future and Emerging Technologies (FET) programme within the Seventh Framework Programme for Research of the European Commission, under FET-Open Grant number 611553.

In this short explanation the algebraic signature $\langle \cdot, \rightarrow, \wedge, \vee, 0, 1 \rangle$ is used for $\mathrm{FL_{ew}}$-algebras, where $\cdot$ stands for the fusion (sometimes also called the multiplicative conjunction or intensional conjunction), $\rightarrow$ for the residuum (also called implication), $\wedge$ for the meet, $\vee$ for the join, 0 for the minimum and 1 for the maximum. The order associated to the lattice operations is denoted by $\leq$. We recall that two famous subvarieties of $\mathbb{FL}_{\mathrm{ew}}$ are the variety $\mathbb{MTL}$ of MTL-algebras [4] and the variety $\mathbb{BL}$ of BL-algebras [5]. The variety $\mathbb{MTL}$ is the subvariety of $\mathbb{FL}_{\mathrm{ew}}$ generated by its chains, and so lately its elements have also been called semilinear $\mathrm{FL_{ew}}$-algebras (e.g., [6]). On the other hand, $\mathbb{BL}$ is the subvariety of $\mathbb{MTL}$ characterized by the following divisibility equation (or identity)

$$\text{(divisibility)} \qquad x \wedge y = x \cdot (x \rightarrow y) \ ,$$

and it is well known to be the subvariety of $\mathbb{MTL}$ generated by continuous t-norms [3]. It is worth saying that while BL-algebras are at present very well understood (see [1] and the recent survey [2]), this is not at all the case neither with MTL-algebras nor with MTL-chains (see [6]).

It is trivial that there are equations (e.g., the very divisibility one) which distinguish

$\mathbb{MTL}$ from $\mathbb{BL}$, i.e., equations which hold in all BL-algebras but fail in some MTL-algebra. In this contribution we want to address this question under the restriction of only allowing equations in the positive fragment. The *positive fragment* is the one given by just considering the operations $\cdot$, $\wedge$, $\vee$, 0 and 1. Thus, the positive fragment does not allow the use of $\to$ (and neither the usual negation $\neg$ nor addition $+$). The terms in the positive fragment will be called *positive terms*; and analogously, *positive equations* refer to equations in the positive fragment. The main problem we are interested is the following:

Problem. Are $\mathbb{MTL}$ and $\mathbb{BL}$ equationally distinguishable in the positive fragment? That is, is there some positive equation which holds in $\mathbb{BL}$ but not in $\mathbb{MTL}$?

The answer to this question is affirmative. Indeed, the following result holds.

Main Theorem. The equation

$$(x_1 \cdot x_4 \cdot x_7) \wedge (x_2 \cdot x_5 \cdot x_8) \wedge (x_3 \cdot x_6 \cdot x_9) \ \leq \ (x_1 \cdot x_2 \cdot x_3) \vee (x_4 \cdot x_5 \cdot x_6) \vee (x_7 \cdot x_8 \cdot x_9)$$

is valid in $\mathbb{BL}$, but fails in $\mathbb{MTL}$.

The failure of this equation in $\mathbb{MTL}$ has been proved by the author exhibiting a concrete counterexample: the 36-element involutive IMTL-chain whose fusion table is shown later in this abstract. It is worth noticing that the size of this chain is too big to be found using a brute-force attack; and indeed, the more interesting part of this research is the explanation of the methodology employed to find this *exotic* MTL-chain.

[1] S. Aguzzoli and S. Bova, *The free n-generated BL-algebra*, ***Annals of Pure and Applied Logic***, vol. 161 (2010), no. 9, pp. 1144–1170.

[2] M. Busaniche and F. Montagna, *Hájek's logic BL and BL-algebras*, ***Handbook of Mathematical Fuzzy Logic. Volume 1*** (P. Cintula, P. Hájek, and C. Noguera, editors), College Publications, 2011, pp. 355–447.

[3] R. Cignoli, F. Esteva, L. Godo, and A. Torrens, *Basic fuzzy logic is the logic of continuous t-norms and their residua*, ***Soft Computing***, vol. 4 (2000), no. 2, pp. 106–112.

[4] F. Esteva and L. Godo, *Monoidal t-norm based logic: towards a logic for left-continuous t-norms*, ***Fuzzy Sets and Systems***, vol. 124 (2001), no. 271, pp. 271–288.

[5] P. Hájek. ***Metamathematics of fuzzy logic***, Kluwer, Dordrecht, 1998.

[6] R. Horčík, *Algebraic semantics: Semilinear FL-algebras*, ***Handbook of Mathematical Fuzzy Logic. Volume 1*** (P. Cintula, P. Hájek, and C. Noguera, editors), College Publications, 2011, pp. 283–353.

Table 1: The Caley table of $\cdot$ in the exotic algebra $\boldsymbol{E}$

```
#FUSION
fus_table = [
  # 0, 1, 2, 3, 4, 5, 6, 7, 8, 9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35      /
  #----------------------------------------------------------------------------------------------------------------/---
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0], # /  0
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1], # /  1
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2], # /  2
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 3, 3], # /  3
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 3, 4], # /  4
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 5, 5, 5, 5, 5], # /  5
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 5, 5, 5, 6], # /  6
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 3, 5, 5, 7, 7, 7], # /  7
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 3, 5, 5, 7, 7, 8], # /  8
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 3, 4, 5, 6, 7, 7, 9], # /  9
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,10,10,10,10,10,10,10,10,10,10], # / 10
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1,10,10,10,10,10,10,10,10,10,11], # / 11
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 3,10,10,10,10,10,10,10,12,12,12], # / 12
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 5, 5, 5,10,10,10,10,10,13,13,13,13,13], # / 13
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 5, 5,10,10,10,10,10,13,13,13,13,14], # / 14
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 5, 5, 5,10,10,10,10,10,13,13,13,14,15], # / 15
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 3, 3, 5, 7, 7,10,10,10,12,12,13,13,16,16,16], # / 16
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 3, 3, 5, 7, 8,10,10,11,12,12,13,13,16,16,17], # / 17
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,18,18,18,18,18,18,18,18,18,18,18,18,18,18,18,18,18,18], # / 18
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1,18,18,18,18,18,18,18,18,18,18,18,18,18,18,18,18,18,19], # / 19
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 3,18,18,18,18,18,18,18,18,18,18,18,18,18,18,18,20,20,20], # / 20
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 3,18,18,18,18,18,18,18,18,18,18,18,18,18,18,18,20,20,21], # / 21
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 3, 3,18,18,18,18,18,18,18,18,18,18,18,18,18,18,18,20,21,22], # / 22
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 5, 5, 5, 5, 5,18,18,18,18,18,18,18,18,18,18,18,18,18,23,23,23,23,23], # / 23
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 5, 5, 5, 7, 7,18,18,18,18,18,18,18,18,18,18,18,20,20,23,23,24,24,24], # / 24
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 5, 5, 5, 7, 8,18,18,18,18,18,18,18,18,18,18,19,20,20,23,23,24,24,25], # / 25
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,10,10,10,10,10,10,10,10,18,18,18,18,18,18,18,18,26,26,26,26,26,26,26,26,26,26], # / 26
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 1,10,10,10,10,10,10,10,10,18,18,18,18,18,18,18,18,26,26,26,26,26,26,26,26,26,27], # / 27
  [ 0, 0, 0, 0, 0, 0, 0, 0, 1, 1,10,10,10,10,10,10,10,11,18,18,18,18,18,18,18,19,26,26,26,26,26,26,26,26,26,28], # / 28
  [ 0, 0, 0, 0, 0, 0, 0, 3, 3, 3,10,10,10,10,10,10,12,12,18,18,18,18,18,18,20,20,26,26,26,26,26,26,26,29,29,29], # / 29
  [ 0, 0, 0, 0, 0, 0, 1, 3, 3, 4,10,10,10,10,10,10,12,12,18,18,18,18,18,18,20,20,26,26,26,26,26,26,27,29,29,30], # / 30
  [ 0, 0, 0, 0, 0, 5, 5, 5, 5, 5,10,10,10,13,13,13,13,13,18,18,18,18,18,23,23,23,26,26,26,26,26,31,31,31,31,31], # / 31
  [ 0, 0, 0, 0, 1, 5, 5, 5, 5, 6,10,10,10,13,13,13,13,13,18,18,18,18,18,23,23,23,26,26,26,26,27,31,31,31,31,32], # / 32
  [ 0, 0, 0, 3, 3, 5, 5, 7, 7, 7,10,10,12,13,13,13,16,16,18,18,20,20,20,23,24,24,26,26,26,29,29,31,31,33,33,33], # / 33
  [ 0, 0, 1, 3, 3, 5, 5, 7, 7, 7,10,10,12,13,13,14,16,16,18,18,20,20,21,23,24,24,26,26,26,29,29,31,31,33,33,34], # / 34
  [ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35], # / 35
]
```