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## Book of Abstracts

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# Gödel algebras with operators and their logics

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Fuzzy modal logic is an active and relatively recent area of research aimed at generalizing classical modal logic to the many-valued or fuzzy framework. This is usually done by considering a Kripke-style relational semantics in which both accessibility relations and evaluations of modal formulas (in each world) are allowed to take values in the real unit interval $[0, 1]$, instead of the classical two-valued set $\{0, 1\}$ (see [3, 4] for instance). In the recent contribution [5] we put forward a new, algebraic-oriented perspective to the area of fuzzy modal logic, and in particular to Gödel modal logic by defining and studying the class of *finite Gödel algebras with operators* (finite *GAOs* for short). These structures are obtained by expanding the language of Gödel algebras by means of two modal operators $\Diamond$ and $\Box$ equationally described by the same axioms used to define these operators in Boolean algebras with operators (BAOs), see [2]: $\Box\top = \top$, $\Diamond\bot = \bot$, $\Box(x \wedge y) = \Box x \wedge \Box y$ and $\Diamond(x \vee y) = \Diamond x \vee \Diamond y$.

The duality between finite Gödel algebras and finite forests (see [1]) leads us to introduce the dual structures of GAOs as triples $(\mathbf{F}, R_\Diamond, R_\Box)$, where $\mathbf{F} = (F, \leq)$ is a finite forest, while $R_\Diamond$ and $R_\Box$ are binary (crisp) relations on $\mathbf{F}$ satisfying suitable conditions of (anti-)monotonicity in their first argument. These structures are called *forest frames*. The main result of [5] shows that, any finite GAO $(\mathbf{A}, \Diamond, \Box)$ uniquely determines a forest frame $(\mathbf{F}_\mathbf{A}, Q_\Diamond, Q_\Box)$ which, in turn, uniquely defines a GAO $(\mathbf{S}_{\mathbf{F}_\mathbf{A}}, \delta_\Diamond, \beta_\Box)$ which is isomorphic to the original one.

The aim of this contribution is to present our representation for finite Gödel algebras with operators and to present further research lines in this direction. In particular, we first discuss on a possible way to extend of the same to the whole family of Gödel algebras with operators (the so defined variety will be denoted by $\mathbb{GA}\mathbb{O}$) and secondly we present a logic, denoted by $\mathbf{GK}_{\Diamond,\Box}$ being its syntactic counterpart. As for the last two proposals:

(1) We introduce the class of *linear Esakia frames* as the relational dual of Gödel algebras with operators. These are obtained by endowing a linear Esakia space $\mathbf{E} = (E, \leq, \tau)$ (i.e., an Esakia space where $(E, \leq)$ is a forest) by two binary relations $R_\Diamond$ and $R_\Box$ satisfying the same properties of forest frames. By using these relational models we will present a generalization of the aforementioned isomorphic representation theorem to the whole class of Gödel algebras with operators.

(2) The logical counterpart of the variety $\mathbb{GAO}$ of Gödel algebras with operators is a system $\mathbf{GK}_{\Diamond,\Box}$ which can be equivalently regarded as the modal intuitionistic logic $\mathbf{IntK}_{\Diamond,\Box}$ (see [6]) added with the prelinearity axioms $(\varphi \to \psi) \vee (\psi \to \varphi)$. We will prove that $\mathbf{GK}_{\Diamond,\Box}$ has the finite model property and, as consequence of the fact that Gödel algebras with operators and linear Esakia frames shares the same tautologies, we will finally show that $\mathbf{GK}_{\Diamond,\Box}$ is sound and complete w.r.t. both these classes of models.

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