# $t$-DeLP: a temporal extension of the defeasible logic programming argumentative framework

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**Abstract.** The aim of this paper is to offer an argumentation-based defeasible logic that enables forward reasoning with time. We extend the DeLP logical framework by associating temporal parameters to literals. A temporal logic program is a set of temporal literals and durative rules. These temporal facts and rules combine to into durative arguments representing temporal processes, that permit us to reason defeasibly about future states. The corresponding notion of logical consequence, or warrant, is defined slightly different from that of DeLP, due to the temporal aspects. As usual, this notion takes care of inconsistencies, and in particular we prove the consistency of any logical program whose strict part is consistent. Finally, we define and study a sub-class of arguments that seem appropriate to reason with natural processes, and suggest a modification to the framework that is equivalent to restricting the logic to this class of arguments.

## 1 Introduction and motivation.

In this contribution, we present a temporal defeasible logic, with temporal literals (for facts) and durative strict or defeasible rules. The main motivation is to encode reasoning about the evolution of processes involving time.

An important feature of *defeasible* logics is the logical parsimony one obtains both at the level of representing knowledge bases (like for the family of non-monotonic logics), as well as regarding the associated logical machinery. This is in accordance with causal reasoning, where it is a standard practice not to list all the conditions involved in a particular process, but only those which are uncommon or specific to this process (e.g. a *spark* is listed among the causes of fire, rather than *the presence of oxygen in air*).

Among non-monotonic logics, those based on an argumentation process present several advantages. First, application of rules is not conditional on any consistency test (in contrast to e.g. default logics); instead, argumentation permits an inconsistency elimination process based on the reasons for and against relevant propositions, *only when* these relevant propositions apply. On the other hand, logical consequence relations built upon arguments are based on a preference relation (between conflicting arguments) which is more modular than the priority relation of purely rule-based approaches. Finally, another advantage of argumentation-based logics is that these mirror the inference mechanisms of a deliberating agent (e.g. a human agent), thus producing logical formalisms that appear more natural and conceptually transparent.

An important contribution along these lines is García and Simari's [5]. The authors present an argumentation-based defeasible logic, called DeLP, and discuss several issues related to application domains. For instance, the question of which criteria the preference relation should be based upon is discussed at length. These criteria play a central role in argumentation-based logics, since they determine which relation of logical consequence one obtains.

In our contribution, though, we show some genuinely novel features, mainly due to the temporal asymmetry (past vs. future) in the descriptions of processes that cannot be modeled within the framework proposed by [5]. With more detail, this asymmetry must be taken into account in the formal criteria the relation of preference is to based upon. As a consequence, the notion of warranted (temporal) literals is slightly different than that studied in [5]. Finally, other phenomena like persistence (and other notions studied in the areas of reasoning with change) are genuinely temporal and demand an explicit treatment within our framework.

Another reason to use defeasible logic, more to the point of the present contribution, is its ability to reason with interactions between different processes or between different aspects of a given process. Typically, a temporal or causal statement describes a class of processes in idealized or isolated conditions, so in practice interactions or influences may exist. Non-trivial interactions, from a logical point of view, can be seen as consisting of different sets of premises with contradicting conclusions.

The temporal argumentation framework we present, $t$-DeLP logic, permits instead to naturally address the problem posed by these contradictions in a compact way. This system does detect and remove all the contradictions that exist according to a given logical program. Moreover this logic can compute the positive facts resulting from these interactions, if the logical program is supplied with sufficient knowledge.

Our proposal, then, is an extension of logic García and Simari's DeLP [5]. We introduce discrete temporal parameters to literals, to form temporal literals denoting the time when the corresponding fact holds. Rules (and arguments) are made of temporal literals and thus encode the delay between each premise and the conclusion. These literals and rules (or arguments) represent, respectively, temporal facts and durative processes.

The paper is structured as follows: after some preliminaries, we present $t$-DeLP logic and study some of its logical properties. In particular we prove that any temporal logical program outputs a consistent set of warranted literals. Finally, we focus on the study of a particularly interesting sub-class of arguments that presumably capture natural processes. We produce a counter-example showing that if we do not restrict to this class some unintuitive consequences may occur. Then we revise some of the definitions to prune these counterexamples, and show that this revised notion of (unrestricted) logical consequence coincides with the relation of logical consequence when restricted to arguments in the class.

**Notation** We will use the following conventions: strong negation is denoted $\sim p$, for a propositional variable $p \in \mathsf{Var}$. Given two sets $X, Y$ we denote the size of $X$ as $|X|$, set-theoretic difference as $X \smallsetminus Y$, the power set of $X$ as $\mathcal{P}(X)$, and the Cartesian product of $X$ and $Y$ as $X \times Y$, or $X^2$ for $X \times X$; $X^{<\omega}$ is the set of finite sequences of elements of $X$. If $f$ is a function $f : X \to Y$ and $X' \subseteq X$, we define $f[X'] = \{f(a) \in Y \mid a \in X'\}$. Given a family of sets $\mathbf{M}$, its union is denoted $\bigcup \mathbf{M}$. If $\sigma$ an expression, $\sigma\binom{\zeta}{\zeta'}$ is the expression obtained by replacing in $\sigma$ each occurrence of $\zeta$ by an occurrence of $\zeta'$.

# 2 Preliminaries

## 2.1 Defeasible logic programming DeLP.

In [5], an argumentation-based defeasible logic was presented. Its language contains literals and rules. Literals are expressions of the form $p$, $\sim p$ from a given set of variables, $p \in \mathsf{Var}$. Strong negation $\sim$ cannot be nested, so we will use the following notation over literals: if $\ell = p$ then $\sim\ell$ will denote $\sim p$, and if $\ell = \sim p$ then $\sim\ell$ will denote $p$. Strict and defeasible rules are expressions $\delta$ of the form $\mathsf{head}(\delta) \leftarrow \mathsf{body}(\delta)$ and (resp.) $\mathsf{head}(\delta) \prec \mathsf{body}(\delta)$, where $\mathsf{head}(\delta)$ is a literal (the conclusion of $\delta$), and $\mathsf{body}(\delta)$ is a finite sequence of literals denoting a conjunction of the conditions for the rule to apply. Strict rules preserve the truth status (be it defeasible or undefeasible) from premises to conclusion while defeasible rules make the conclusion defeasibly true given its premises are true. Thus in particular, the conclusions of arguments making use of strict information only cannot be withdrawn.

DeLP's relation of logical consequence relation, called warrant, defines a consistent subset of the set of derivable literals (the latter being typically inconsistent for a given knowledge base). Warrant is defined in terms of the interactions between conflicting arguments, according to some preference or defeat relation between these arguments. The authors of [5] propose formal criteria based on more direct rules[1] and more premises.

The present contribution, focusing on temporal reasoning, favors a reading of defeasible rules based on the temporal asymmetry between causes and effects[2]. This requires a different defeat relation between arguments, i.e. one based on different criteria; and, more importantly, a different notion of attack and argumentation line (since future events cannot contradict events that already took place). Thus, the corresponding relation of logical consequence is slightly different to that presented in [5].

## 2.2 Knowledge Representation: temporal predicates and constraints.

The present results are stated at the propositional level, for a set of literals (with associated time). In order to reason with richer representations, though, we may rather see these literals as expressing ground predicates, i.e. with *literal* expressions encoding something of the form *literal = (object property)* or also *literal = (object, parameter, value)*; temporal literals, then, are represented as a pair: *(literal, time)*.

Time, then, is also relevant to determine whether a pair of expressions (each denoting an event) contradict each other: for this contradiction to exist, the literals expressed must be the negation of each other*and* they must be claimed to hold at the same time. Moreover, our framework permits to express a temporal or causal statement (possibly an instance of some general law) as a rule: *a set of tuples (object, parameter, value,*

[1] As exemplified by the set of rules { *birds fly*, *penguins are birds*, *penguins do not fly* } .

[2] In this sense, it contrasts with the more general, evidence-based reasoning that usually motivates non-monotonic reasoning.

*time)* imply *a tuple (object, parameter, value, time)*. Among arguments (combinations of facts and rules), those with positive duration express how (an aspect of) some process does change with time, while those with no duration may be used to model static or structural constraints.

In both cases, when a literal represents something of the form *(object, parameter, value, time)*, it is convenient to represent the constraints that occur in the represented expression, e.g., that an object cannot have different values of a given parameter at a given time. These constraints can be represented by strict rules (if absolute) or defeasible rules (if contingent), induced by a set of such mutually exclusive propositions capturing these constraints: such a set $X$ induces rules of the form $\sim\ell \leftarrow \ell'$ or $\sim\ell \prec\ell'$, where $\ell, \ell'$ are arbitrary elements of $X$ with $\ell \neq \ell'$.

For an example of absolute constraints, let $\mathcal{O}$ and $\mathcal{L}$ be the sets of objects $o$ and locations $l$; and let $@(o, l) \in \mathsf{Var}$ denote: *$o$ is at $l$*. Then,

- the *at most one location per object* policy is defined by a set $\{o\} \times \mathcal{L}$ for each $o \in \mathcal{O}$; this set induces rules of the form $\sim\langle @(o, l), t\rangle \leftarrow \langle @(o, l'), t\rangle$, if $l \neq l'$.
- the *at most one object per location* policy is defined by a set $\mathcal{O} \times \{l\}$ for each $l \in \mathcal{L}$; this set induces rules $\langle \sim @(o, l), t\rangle \leftarrow \langle @(o', l), t\rangle$, if $o \neq o'$.

## 3 *t*-DeLP: defeasible logic with (discrete) time.

We take the set of natural numbers $\mathbb{N}$ as our working set of discrete time points. The logic $t$-DeLP is based on temporal literals $\langle \ell, t\rangle$, where $\ell$ is a literal and $t \in \mathbb{N}$, denoting $\ell$ holds at $t$. In order to solve conflicts between arguments the preference (or defeat) relation between arguments is based on: a preference for arguments with *more premises* and for *lengthier* arguments over its parts (so an argument can defeat the persistence of its subarguments' conclusions). Arguments that make only use of strict information are also preferred to arguments conflicting with them. A final criterion, *less durative rules*, is not considered here.[3]

**Definition 1.** *Given a finite set of propositional variables* $\mathsf{Var}$*, we define* $\mathsf{Lit} = \mathsf{Var} \cup \{\sim p \mid p \in \mathsf{Var}\}$*. The define set of temporal literals* $\mathsf{TLit} = \{\langle \ell, t\rangle \mid \ell \in \mathsf{Lit}, t \in \mathbb{N}\}$*. If* $\Gamma \subseteq \mathsf{Lit}$*, we say that* $\Gamma$ *is* consistent *if there is no* $p \in \mathsf{Var}$ *such that* $p, \sim p \in \Gamma$*. If* $\Gamma^* \subseteq \mathsf{TLit}$*, we say that* $\Gamma^*$ *is* consistent *if each* $\Gamma^*_t := \{\ell \mid \langle \ell, t\rangle \in \Gamma^*\}$ *is consistent.*

**Definition 2.** *A* temporal strict (resp. defeasible rule) *is an expression* $\delta$ *of the form* $\langle \ell, t\rangle \leftarrow \langle \ell_0, t_0\rangle, \ldots, \langle \ell_n, t_n\rangle$ *(resp.* $\langle \ell, t\rangle \prec \langle \ell_0, t_0\rangle, \ldots, \langle \ell_n, t_n\rangle$*), where* $t \geq \max\{t_0, \ldots t_n\}$*. We write* $\mathsf{head}(\delta) = \langle \ell, t\rangle$*,* $\mathsf{body}(\delta) = \{\langle \ell_0, t_0\rangle, \ldots, \langle \ell_n, t_n\rangle\}$ *and* $\mathsf{literals}(\delta) = \{\mathsf{head}(\delta)\} \cup \mathsf{body}(\delta)$.

A rule with an empty body, e.g. $\langle \ell, t\rangle \leftarrow$, also denoted $\langle \ell, t\rangle$, (resp. $\langle \ell, t\rangle \prec$ for the defeasible case) represents a basic fact that holds at time $t$ (resp. a *presumable* fact holding at $t$). As in DeLP, a strict rule $\delta \in \Pi$ states the conclusion $\mathsf{head}(\delta)$ is *as true*

[3] This is important, since rules with long duration might fail to detect conflicts, (so, e.g. balls running into each other would magically not collide). Instead, we will assume rules are precise enough.

*as* its premises are. A defeasible rule $\delta \in \Delta$ states a weaker claim: if the premises are true this is in principle a reason for believing that the conclusion is also true (though this conclusion may be withdrawn for other reasons). A special subset of rules is that of *persistence* rules, of the form $\langle \ell, t+1 \rangle \leftarrow \langle \ell, t \rangle$ or $\langle \ell, t+1 \rangle \multimap \langle \ell, t \rangle$, stating that $\ell$ is preserved from $t$ to $t+1$ (if true at $t$) and, resp., that *ceteris paribus* a literal $p$ that holds at $t$ will persist at $t+1$. The set of defeasible (strict) persistence rules will be denoted $\Delta_{\mathsf{p}}$ (resp. $\Pi_{\mathsf{p}}$).

*Example 1.* We saw above an example of a (strict) rule without delay: the spatial constraints $\langle \sim @(o, l'), t \rangle \leftarrow \langle @(o, l), t \rangle$. Among (strict) rules with delay, we may have rules like $\langle \sim \text{tuesday}, t+24 \rangle \leftarrow \langle \text{tuesday}, t \rangle$ and $\langle \text{wednesday}, t+24 \rangle \leftarrow \langle \text{tuesday}, t \rangle$, where each time unit represents an hour.

**Definition 3.** *A* temporal DeLP program*, or t-DeLP program, is a pair* $(\Pi, \Delta)$*, where* $\Pi$ *is a set of temporal strict rules,* $\Delta$ *a set of temporal defeasible rules and the set of derivable literals from* $\Pi$ *is consistent.*

Temporal rules as above can be seen as instances of general rules of the form $\delta^* = \ell \leftarrow (\ell_0, d_0), \ldots, (\ell_n, d_n)$ -and similarly for defeasible rules with $\multimap$ -, where each $d_i$ expresses how much time in advance must $\ell_i$ hold for the rule to apply and produce a derivation of $\ell$. Such a general rule is to be understood as a shorthand for the set of rules $\{\langle \ell, t \rangle \leftarrow \langle \ell_0, t - d_0 \rangle, \ldots, \langle \ell_n, t - d_n \rangle \mid t \in \mathbb{N}, t \geq \max\{d_0, \ldots, d_n\}\}$. For example, the rule $\langle p, 4 \rangle \multimap \langle q, 3 \rangle$ would be an instance of the general rule $p \multimap (q, 1)$. Persistence rules can therefore be expressed as general rules of the form $\ell \leftarrow (\ell, 1)$ or $\ell \multimap (\ell, 1)$; the latter defeasible general persistence rule for $\ell$ will be denoted $\delta_{\{}\ell\}$. The formal definitions do make use only of instances of general rules, i.e. temporal rules only.

*Example 2.* Consider the following example. Lars, a tourist visiting the Snake Forest, has been bitten by a venomous snake. The poison of this type of snake does kill a person in 3 hours. But since our subject, Lars, is experienced (it has been bitten and cured a few times before), he may resist up to 5 hours. We decide to take him to the nearest hospital, which in normal conditions this would take 2 hours, but since today is sunday, the traffic jam makes it impossible to reach the hospital in less than 4 hours. The antidote takes less than an hour to become effective. This scenario is modeled by the following temporal facts and general rules:

$\Pi = \{\langle @\text{snake.forest(Lars)}, 0 \rangle, \langle \text{snake.bitten(Lars)}, 0 \rangle, \langle \text{experienced(Lars)}, 0 \rangle,$
$\quad \langle \sim dead(Lars), 0 \rangle, \langle \text{sunday}, 0 \rangle\},$

$\Delta = \{\text{dead(Lars)} \multimap (\text{snake.bitten(Lars)}, 3)$
$\quad \sim\text{dead(Lars)} \multimap (\text{snake.bitten(Lars)}, 3), (\text{experienced(Lars)}, 3)$
$\quad \text{dead(Lars)} \multimap (\text{snake.bitten(Lars)}, 5), (\text{experienced(Lars)}, 5)$
$\quad @\text{hospital(Lars)} \multimap (\text{snake.bitten(Lars)}, 2), (@\text{snake.forest(Lars)}, 2)$
$\quad \sim @\text{hospital(Lars)} \multimap (\text{traffic.jam}, 2), (\text{snake.bitten(Lars)}, 2), (@\text{snake.forest(Lars)}, 2)$
$\quad @\text{hospital(Lars)} \multimap (\text{traffic.jam}, 4), (\text{snake.bitten(Lars)}, 4), (@\text{snake.forest(Lars)}, 4)$
$\quad \text{traffic.jam} \multimap (\text{sunday}, 0)$
$\quad \sim\text{dead}(Lars) \multimap @(\text{hospital(Lars)}, 1), (\text{snake.bitten(Lars)}, 1), (\sim\text{dead(Lars)}, 1)\ \}$

We also add to $\Delta$ all persistence rules for literals occurring in $\Pi$ (these strict facts only persist defeasibly) or any of the previous rules in $\Delta$. With this information, it can be proved that Lars survives this snake attack.

Derivability in t-DeLP, as in DeLP, is defined by closure under the *modus ponens* rule, and will be denoted by $\vdash$. As it happens in DeLP, the set of derivable literals in $(\Pi, \Delta)$ will not in general be consistent.

*Example 3.* (Cont'd) Consider for instance, the conflict between (a) the derivation of $\langle\sim$ @hospital(Lars)$, 2\rangle$ by means of persistence rules for the literal @snake.forest(Lars) and (b) the derivation of $\langle$@hospital(Lars)$, 4\rangle$, which takes into account that today is sunday. Or, the inconsistency between the latter and (c) the derivation $\langle\sim$@hospital(Lars)$, 2\rangle$, that does not pay attention to the fact that today is *sunday*. The latter conflict is represented in Figure 2.

**Definition 4.** *Given a t-DeLP $(\Pi, \Delta)$, an* argument *for $\langle \ell, t\rangle$ is a set $\mathcal{A} \subseteq \Pi \cup \Delta$, such that*

*(1) $\mathcal{A} \vdash \langle \ell, t\rangle$,*
*(2) the set of derivable literals from $\Pi \cup \mathcal{A}$ is consistent,*
*(3) $\mathcal{A}$ is $\subseteq$-minimal satisfying (1) and (2).*

Observe that, although $\Pi$ and $\Delta$ may be infinite (due to the coding of general rules as an infinite set of temporal rules), an argument for a t-DeLP program $(\Pi, \Delta)$ will be always a finite subset of $\Pi \cup \Delta$.

We also define for an argument $\mathcal{A}$ for $\langle \ell, t\rangle$:

$$\mathsf{concl}(\mathcal{A}) = \langle \ell, t\rangle \qquad \mathsf{base}(\mathcal{A}) = \{\delta \in \mathcal{A} \mid \mathsf{body}(\delta) = \emptyset\}$$
$$\mathsf{literals}(\mathcal{A}) = (\bigcup \mathsf{body}[\mathcal{A}]) \cup \mathsf{head}[\mathcal{A}] \qquad \|\mathcal{A}\| = t - t_0$$

where $t_0 = \min\{t' \in \mathbb{N} \mid \langle \ell', t'\rangle \in \mathsf{base}(\mathcal{A})\}$ and we define $\mathbf{t}(\mathcal{A}) = t_0$. Note that $t = \max\{t' \in \mathbb{N} \mid \langle \ell', t'\rangle \in \mathsf{literals}(\mathcal{A})\}$.

In contrast to DeLP, we make explicit in argument $\mathcal{A}$ which is the strict information used, to facilitate the detection of inconsistencies with an intermediate step in the strict part of argument $\mathcal{A}$. The reason is that there exist many possible ways to complete defeasible rules in $\mathcal{A}$ into a derivation for $\mathsf{concl}(\mathcal{A})$. And these different ways may be attacked by different arguments. For example, the sets $\{\langle p, 4\rangle \leftarrow \langle q, 2\rangle, \langle q, 2\rangle \leftarrow \langle r, 1\rangle, \langle r', 0\rangle\}$ and $\{\langle p, 4\rangle \leftarrow \langle s, 3\rangle, \langle s, 3\rangle \leftarrow \langle r, 1\rangle, \langle r', \rangle\}$ may both complete the set $\{\langle p', 5\rangle \multimap \langle p, 4\rangle, \langle r, 1\rangle \multimap \langle r', 0\rangle\} \subseteq \Delta$ into an argument (derivation) for $\langle p', 5\rangle$, but only the latter is attacked by an argument concluding $\langle \sim s, 3\rangle$.

Now we define a sub-argument of $\mathcal{A}$. A sub-argument will be the actual target of an attack by another argument.

**Definition 5.** *Let $(\Pi, \Delta)$ be a t-DeLP program and let $\mathcal{A}$ be an argument for $\langle \ell, t\rangle$ in $(\Pi, \Delta)$. Given some $\langle \ell_0, t_0\rangle \in \mathsf{literals}(\mathcal{A})$, a* sub-argument *for $\langle \ell_0, t_0\rangle$ is a subset $\mathcal{B} \subseteq \mathcal{A}$ such that $\mathcal{B}$ is an argument for $\langle \ell_0, t_0\rangle$.*

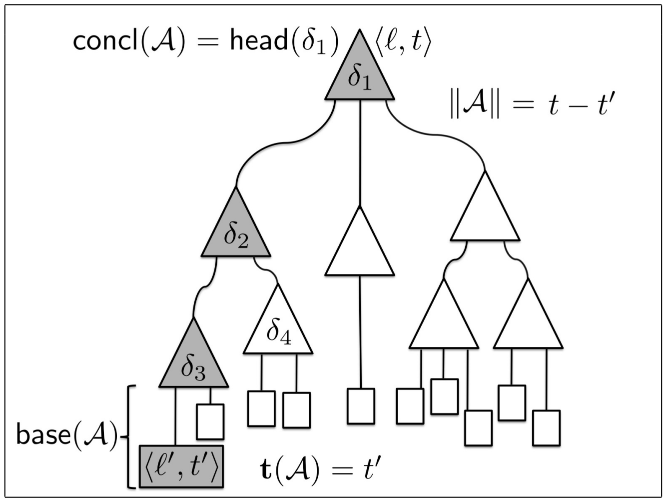


**Fig. 1.** An argument $\mathcal{A}$ and its total duration $\|\mathcal{A}\|$.

Notice that each literal $\langle \ell_0, t_0 \rangle$ in an argument $\mathcal{A}$ uniquely determines its corresponding subargument, that we will denote by $\mathcal{A}(\langle \ell_0, t_0 \rangle)$. For example, in Figure 1, $\mathcal{A}(\mathsf{head}(\delta_2)) = \{\delta_2, \delta_3, \delta_4, \langle \ell', t' \rangle, \ldots\}$.

Let us come back to Example 3 where we have several conflicting conclusions, not all of them being equally preferred. Indeed (b) is to be preferred to both (a) and (c). In the former, we prefer to use new (strict) information rather than mere (defeasible) persistence; in the latter, we prefer to use as much information as possible. To obtain this intuitive preference relations among derivations, we consider the following attack and defeat relations between arguments encoding the corresponding derivations.

**Definition 6.** *Given t-DeLP program* $(\Pi, \Delta)$*, let* $\mathcal{A}_0$ *an argument for* $\langle \ell_0, t_0 \rangle$ *and let* $\mathcal{A}_1$ *an argument for* $\langle \ell_1, t_1 \rangle$*. We say* $\mathcal{A}_1$ attacks $\mathcal{A}_0$ *if there exists a subargument* $\mathcal{B}$ *of* $\mathcal{A}_0$ *for* $\langle \sim\ell, t_1 \rangle$ *and* $\Delta \cap \mathcal{B} \neq \emptyset$*. In this case we say that* $\mathcal{A}_1$ *attacks* $\mathcal{A}_0$ *at* $\mathcal{B}$*.*

Notice that if $\mathcal{A}_1$ attacks $\mathcal{A}_0$ at $\mathcal{B}$, $\mathcal{B}$ cannot only consist of strict information, in particular of a strict fact: $\mathcal{B} \neq \{\langle \ell, t' \rangle\}$.

As in DeLP, one has a further defeat relation to decide which argument prevails in case of an attack. This relation can be in principle specified by the user, but in this paper we adopt the following definition in order to meet the above intuitive preferences in Example 3.

**Definition 7.** *Let* $\mathcal{A}_1$ *attack* $\mathcal{A}_0$ *at* $\mathcal{B}$*, where* $\mathsf{concl}(\mathcal{B}) = \langle \ell, t \rangle$*. We say:*

- $\mathcal{A}_1$ *is a* blocking defeater for $\mathcal{A}_0$ *iff* $\mathsf{base}(\mathcal{A}_1) \not\subseteq \mathsf{base}(\mathcal{B})$ *or* $\mathsf{base}(\mathcal{A}_1) = \mathsf{base}(\mathcal{B})$
- $\mathcal{A}_1$ *is a* proper defeater for $\mathcal{A}_0$ *iff* $\mathcal{A}_1 \subseteq \Pi$*, or* $\mathsf{base}(\mathcal{A}_1) \supsetneq \mathsf{base}(\mathcal{B})$*, or* *for some* $t' < t$*,* $\mathcal{A}_0 = \mathcal{A}_1(\langle \ell, t' \rangle) \cup \{\langle \ell, t''+1 \rangle \prec \langle \ell, t'' \rangle \mid t' \leq t'' < t\}$

*Blocking and proper defeat relations are denoted, resp.,* $\mathcal{A}_1 \prec\succ \mathcal{A}_0$*, and* $\mathcal{A}_1 \succ \mathcal{A}_0$*.*

Thus, a strict set of rules is a proper defeater for any argument attacked by it. In the other cases, a properly defeated argument $\mathcal{A}_0$ either has *less premises than* or is a

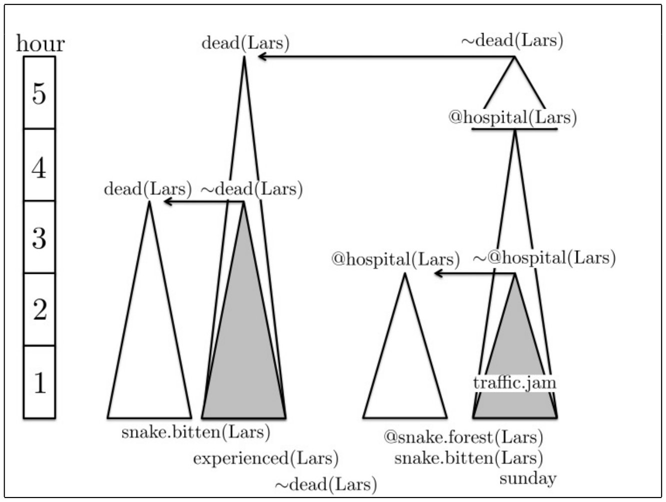


**Fig. 2.** The Snake Bites Lars scenario.

*sub-argument of* its defeater $\mathcal{A}_1$, *extended with* a sequence of $\|\mathcal{A}_1\| - \|\mathcal{B}\|$ instances of persistence rule $\delta_\ell$. In the latter case we say that $\mathcal{A}_1$ is a *lenghtier* argument than the subargument $\mathcal{B}$ of $\mathcal{A}_0$. Observe an argument $\mathcal{A}_1$ does not defeat its sub-argument, only the $\delta_\ell$-extension of it. (See Figure 3 (top left) for an example.) Finally, note that since $\Pi$ is assumed to produce a consistent set of derivable literals and the other two conditions for being a proper defeater are asymmetric relations, no pair of arguments can be a proper defeater for each other.

An argument $\mathcal{B}$ defeating $\mathcal{A}$ can at its turn have its own defeaters $\mathcal{C}, \ldots$ and so on. These give rise to *argumentation lines* where each argument defeats its predecessor. Intuitively, the notion of defeat in an argumentation line $[\ldots, \mathcal{A}, \mathcal{B}, \mathcal{C}, \ldots]$ should exclude a blocking defeater $\mathcal{C}$ for $\mathcal{B}$ as a defeater, *provided that* $\mathcal{B}$ is already blocking defeater for $\mathcal{A}$. (The reason is that otherwise, we could have cycles $[\ldots, \mathcal{A}, \mathcal{B}, \mathcal{A}, \mathcal{B}, \ldots]$.) Other forms of cyclic defeats are also excluded in the definition. Note condition (ii) is slightly weaker than its DeLP counterpart in [5].

**Definition 8.** *(Adapted from [5]) Let $\mathcal{A}_1$ be an argument in $(\Pi, \Delta)$. An* argumentation line *for $\mathcal{A}_1$ is a sequence $\Lambda = [\mathcal{A}_1, \mathcal{A}_2, \ldots]$ where*

*(i) supporting arguments, i.e. in odd positions $\mathcal{A}_{2i+1} \in \Lambda$ are jointly $\mathbf{M}$-consistent, and similarly for interfering arguments $\mathcal{A}_{2i} \in \Lambda$*
*(ii) a sub-argument of $\mathcal{A}_i$ can occur later in $\Lambda$, i.e. as $\mathcal{A}_{i+2j}$ only if $\|\mathcal{A}_{i+2j}\| < \|\mathcal{A}_i\|$ (i.e. its duration is stricly less than that of $\mathcal{A}_i$)*[4]
*(iii) $\mathcal{A}_{i+1}$ is a proper defeater for $\mathcal{A}_i$ if $\mathcal{A}_i$ is a blocking defeater for $\mathcal{A}_{i-1}$*

[4] This is a weaker condition that in DeLP, where no sub-argument *at all* can occur later than an argument in $\Lambda$. In our temporal case, a sub-argument (of $\mathcal{A}$) talking about a previous time may offer legitimate reasons to the defense of $\mathcal{A}$.

*The union of maximal argumentation lines $\Lambda$s for $\mathcal{A}_1$, under the defeat relation $\succ \cup \prec\succ$ constrained by $\Lambda$, arranged in the form of a tree*[5]*, is the so-called* dialectical tree *for $\mathcal{A}_1$:*

$$\mathcal{T}_{(\Pi,\Delta)}(\mathcal{A}_1) = \bigcup\{\Lambda \in (\Pi \cup \Delta)^{<\omega} \mid \Lambda \textit{ is a maximal arg. line for } \mathcal{A}_1\}$$

The next bottom-up marking procedure on the tree $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{A}_1)$ decides whether $\mathcal{A}_1$ is undefeated in $(\Pi, \Delta)$.

**Definition 9.** *[5] Let $\mathcal{T} = \mathcal{T}_{(\Pi,\Delta)}(\mathcal{A}_1)$ be the dialectical tree for $\mathcal{A}_1$. Then,*

*(1) mark all terminal nodes of $\mathcal{T}$ with a $U$ (for undefeated);*
*(2) mark a node $\mathcal{B}$ with a $D$ (for defeated) if it has a children node marked $U$;*
*(3) mark $\mathcal{B}$ with $U$ if all its children nodes are marked $D$.*

See Figure 3 (right) for an example of a dialectical tree with root $\mathcal{A}_1$. Arguments marked $U$ are represented white, and those marked $D$ are represented black. (Similarly for the arrows: an arrow between a $U$ element responsible for its parent node being marked $D$ is painted white; otherwise it is black).

**Definition 10.** *Given a t-DeLP program $(\Pi, \Delta)$, we say $\langle \ell, t\rangle$ is* warranted *in $(\Pi, \Delta)$ iff there exists an argument $\mathcal{A}_1$ for $\langle \ell, t\rangle$ in $(\Pi, \Delta)$ such that $\mathcal{A}_1$ is undefeated in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{A}_1)$. We will denote by $\mathsf{warr}(\Pi, \Delta)$ the set of warranted literals in $(\Pi, \Delta)$.*

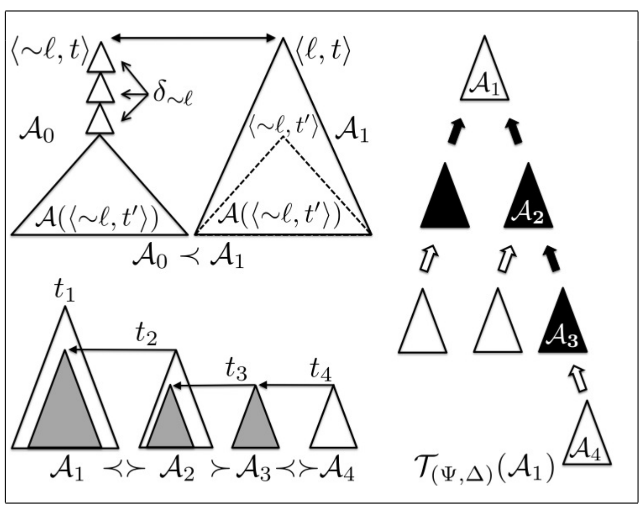


**Fig. 3.** (Top Left) A proper defeater. (Bottom Left) An argumentation line. (Right) The dialectical tree $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{A}_1)$.

The next results show $t$-DeLP ensures the consistency of a $t$-DeLP logical program $(\Pi, \Delta)$, provided that its strict part $\Pi$ outputs a consistent set of derivable literals.

[5] Where all paths from the root to the leaf nodes exactly correspond to all the possible maximal argumentation lines.

**Lemma 1.** *Given some $(\Pi, \Delta)$, let $\mathcal{A}$ be an argument in $(\Pi, \Delta)$ for $\langle \ell, t\rangle$. Also, let $\mathcal{B}$ be an argument for $\langle \sim\ell, t\rangle$, with $\mathcal{A}$ a defeater for $\mathcal{B}$ at $\mathcal{B}$. If $\mathcal{A}$ is defeated in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{B})$, then $\mathcal{A}$ is defeated in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{A})$.*

*Proof.* Let $\mathcal{A}$ a defeater for $\mathcal{B}$ at $\mathcal{B}$. This implies the existence of some $\Lambda = [\mathcal{B}, \mathcal{A}, \ldots] \subseteq \mathcal{T}_{(\Pi,\Delta)}(\mathcal{B})$. Assuming $\mathcal{A}$ is defeated in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{B})$, we have in particular some $\Lambda^* = [\mathcal{B}, \mathcal{A}, \mathcal{B}_3, \ldots, \mathcal{B}_{2m+1}]$ witnessing the defeat of $\mathcal{A}$ (i.e. with $\mathcal{B}_{2i+1}$ undefeated in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{B})$, for each $1 \leq i \leq m$).

We show first that any arg. line $[\mathcal{B}, \mathcal{A}, \mathcal{B}_3, \ldots]$ in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{B})$ contains a sequence $[\mathcal{A}, \mathcal{B}_3, \ldots]$ that is an arg. line in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{A})$. Let then $[\mathcal{B}, \mathcal{A}, \mathcal{B}_3, \ldots] \subseteq \mathcal{T}_{(\Pi,\Delta)}(\mathcal{B})$. Clearly $\Lambda = [\mathcal{A}, \mathcal{B}_3, \ldots]$ satisfies the 3 conditions for an arg. line for $\mathcal{A}$: (1) its first element is $\mathcal{A}$; (2) the set of even (odd) members is jointly consistent (since, otherwise, the set of odd (resp. even) members of $[\mathcal{B}, \mathcal{A}, \mathcal{B}_3, \ \ldots]$ would also be inconsistent). (3) a sub-argument of some argument $\mathcal{B}_i$ (with the same duration than $\mathcal{B}_i$) does not occur after argument $\mathcal{B}_i$ in the line (otherwise, the same would be true of $[\mathcal{B}, \mathcal{A}, \mathcal{B}_3, \ldots]$ in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{B})$). Finally, (4) for no three consecutive elements $[\ldots, \mathcal{D}_i, \mathcal{D}_{i+1}, \mathcal{D}_{i+2}, \ldots]$ we have $\mathcal{D}_{i+2}$ is a blocking defeater for $\mathcal{D}_{i+1}$ and $\mathcal{D}_{i+1}$ a blocking defeater for $\mathcal{D}_i$ (since otherwise the same would occur in arg. line $[\mathcal{B}, \mathcal{A}, \mathcal{B}_3, \ \ldots]$).

Now, assume, towards a contradiction, that $\mathcal{A}$ is undefeated in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{A})$. We show the previous inclusion, namely that any $[\mathcal{B}, \mathcal{A}, \ldots] \subseteq \mathcal{T}_{(\Pi,\Delta)}(\mathcal{B})$ is such that $[\mathcal{A}, \ldots] \subseteq \mathcal{T}_{(\Pi,\Delta)}(\mathcal{A})$, plus both assumptions ($\mathcal{A}$ is defeated in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{B})$ and $\mathcal{A}$ is undefeated in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{A})$) imply the existence of an increasing sequence of argumentation lines of arbitrarily finite length, which is impossible.

The previous inclusion shows in particular that witness $\Lambda^*$-minus-$\mathcal{B}$ is an arg. line in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{A})$. Since $\mathcal{A}$ is undefeated in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{A})$, some $\mathcal{B}_3$ must be defeated in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{A})$. Let $\Lambda_0 = [\mathcal{A}, \mathcal{B}_3, \mathcal{C}_1^0, \ldots, \mathcal{C}_{2n_0+1}^0]$ witness the defeat of $\mathcal{B}_3$ in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{A})$; i.e. $\mathcal{C}_{2n'_0+1}^0$ is undefeated in this tree, for any $n'_0 \leq n_0$. By assumption on the original witness $\Lambda^*$ in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{B})$, if $\mathcal{C}_1^0$ occurs in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{B})$, then $\mathcal{C}_1^0$ must be defeated in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{B})$. To see this $\mathcal{C}_1^0$ will effectively occur in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{B})$ it suffices to prove $\mathcal{C}_1^0$ is not a sub-argument of $\mathcal{B}$ with $\|\mathcal{C}_1^0\| = \|\mathcal{B}\|$. For this, assume the contrary. Then, by Def. of arg. line, we have $\|\mathcal{B}\| = \|\mathcal{A}\| = \|\mathcal{B}_3\| = \|\mathcal{C}_1^0\|$. But then, $\mathcal{C}_1^0 = \mathcal{B}(\mathsf{concl}(\mathcal{C}_1^0))$, $\|\mathcal{C}_1^0\| = \|\mathcal{B}\|$ and $\mathcal{C}_1^0$ a defeater for $\mathcal{B}_3$ (hence inconsistent with it) jointly imply that $\mathcal{B}$ and $\mathcal{B}_3$ are not consistent (contradiction). Moreover, this $\mathcal{C}_1^0$ satisfies in the tree for $\mathcal{B}$ the restriction against two consecutive blocking defeaters, since it satisfies this restriction in the tree for $\mathcal{A}$ (this preservation is automatic since $\mathcal{C}_1^0$ is not the second element in $\Lambda_0$).

Let then $\Lambda_1 = [\mathcal{B}, \mathcal{A}, \mathcal{B}_3, \mathcal{C}_1^0, \mathcal{C}_1^1, \ldots, \mathcal{C}_{2n_1+1}^1]$ be a witness to the defeat of $\mathcal{C}_1^0$. By the former inclusion, this latter witness $\Lambda_1$-minus-$\mathcal{B}$ is in the tree for $\mathcal{A}$. By the assumption that $\mathcal{A}$ is undefeated in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{A})$, the element $\mathcal{C}_1^1$ of this witness must be defeated in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{A})$, since $\mathcal{C}_1^0$ is undefeated in it. Let $\Lambda_2 = [\mathcal{B}, \mathcal{A}, \mathcal{B}_3, \mathcal{C}_1^0, \mathcal{C}_1^1, \mathcal{C}_1^2 \ldots, \mathcal{C}_{2n_2+1}^2]$ be a witness to the defeat of $\mathcal{C}_1^1$.

This procedure can be continued *ad infinitum* with analogous reasonings from $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{B})$ to $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{A})$ and viceversa. Thus, there exists an infinite sequence of arg. lines (witnesses) $\Lambda_n$ of the form $[\mathcal{B}, \mathcal{A}, \mathcal{B}_3, \mathcal{C}_1^0, \mathcal{C}_1^1, \ldots, \mathcal{C}_1^n, \ldots]$ (for $n$ odd) or of the form $[\mathcal{A}, \mathcal{B}_3, \mathcal{C}_1^0, \mathcal{C}_1^1, \ldots, \mathcal{C}_1^n, \ldots]$ (for $n$ even). Thus, arg. lines of arbitrarily finite length must exist, and elements of the form $\mathcal{C}_1^n$ form an infinite sequence $\Lambda_\omega = [\mathcal{A}, \mathcal{B}_3, \mathcal{C}_1^0, \mathcal{C}_1^1, \ldots, \mathcal{C}_1^n, \mathcal{C}_1^{n+1}, \ldots]$ satisfying: any initial segment of $\Lambda_\omega$ is an arg. line.

We show such an infinite sequence $\Lambda_\omega$ cannot exist. Since $\mathbf{t}(\mathcal{A}) + \|\mathcal{A}\|$ is finite, and arguments $\mathcal{C}$ in $\Lambda_\omega$ must satisfy $\|\mathcal{C}\| \leq \|\mathcal{A}\|$, we have that rules in these arguments $\mathcal{C}$ are finite sequences of literals in the finite set $\mathsf{Lit} \times \{0, \ldots, \mathbf{t}(\mathcal{A}) + \|\mathcal{A}\|\}$. Hence, the number of these rules is finite. Hence, there are only finitely many different arguments which can occur in $\Lambda_\omega$. But since $\Lambda_\omega$ is infinite, we will have some repetition $\mathcal{C}_1^j = \mathcal{C}_1^{j+i}$. Then, the sequence $[\mathcal{A}, \mathcal{B}_3, \ldots, \mathcal{C}_1^j, \ldots, \mathcal{C}_1^{j+i}]$ will violate the corresponding condition of Definition 8. Thus, such an infinite sequence cannot exist (contradiction). We infer that $\mathcal{A}$ must also be defeated in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{A})$, provided it is defeated in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{B})$. □

**Theorem 1.** *Given a t-de.l.p.* $(\Pi, \Delta)$*, the set of literals* $\mathsf{warr}(\Pi, \Delta)$ *is consistent.* [6]

*Proof.* Let $\langle \ell, t \rangle \in \mathsf{warr}(\Pi, \Delta)$. Thus, some argument $\mathcal{A}$ for $\langle \ell, t \rangle$ in $(\Pi, \Delta)$ exists that is undefeated in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{A})$. It suffices to show that $\langle \sim\ell, t \rangle \notin \mathsf{warr}(\Pi, \Delta)$. The reason is that if, instead, an attack occurred at a previous time, i.e. $\mathcal{A}$ was attacked at some $\mathcal{A}(\langle \ell_0, t_0 \rangle)$, and defeated by some $\mathcal{B}$, the same reasoning given next would apply for $\mathcal{A}(\langle \ell_0, t_0 \rangle)$ and the corresponding defeater $\mathcal{B}$ (i.e. that $\langle \sim\ell_0, t_0 \rangle \notin \mathsf{warr}(\Pi, \Delta)$.

Thus, assume -towards a contradiction- that $\langle \sim\ell, t \rangle \in \mathsf{warr}(\Pi, \Delta)$. Then some argument $\mathcal{B}$ for $\langle \sim\ell, t \rangle$ exists in $(\Pi, \Delta)$, undefeated in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{B})$. Observe first that if $\mathcal{A} \subseteq \Pi$, then either each such argument $\mathcal{B}$ contains some rule in $\Delta$, in which case $\mathcal{A}$ will attack and defeat any such $\mathcal{B}$ (contradicting that $\langle \sim\ell_0, t_0 \rangle \in \mathsf{warr}(\Pi, \Delta)$); or, also some such $\mathcal{B}$ for $\langle \sim\ell_0, t_0 \rangle$ is a subset of $\Pi$, contradicting the assumption that the set of derivable literals from $\Pi$ alone is consistent. Thus, we may assume that $\mathcal{A} \cap \Delta \neq \emptyset$. Now, consider again the possibility that some such argument $\mathcal{B}$ for $\langle \sim\ell_0, t_0 \rangle$ is a subset of $\Pi$. Then, $\mathcal{A}$ is defeated by an (unattacked, hence) undefeated argument, contradicting the initial assumption $\langle \ell, t \rangle \in \mathsf{warr}(\Pi, \Delta)$. Thus, we may assume that both $\mathcal{A}$ and $\mathcal{B}$ contain some defeasible rule.

This implies that $\mathcal{A}$ attacks $\mathcal{B}$ at $\mathcal{B}$, and $\mathcal{B}$ attacks $\mathcal{A}$ at $\mathcal{A}$. Consider next the following cases. (Case) $\mathcal{A}$ is not a defeater for $\mathcal{B}$. Then, since the only possibilities are $\mathsf{base}(\mathcal{A}) \not\subseteq \mathsf{base}(\mathcal{B})$ and $\mathsf{base}(\mathcal{A}) \neq \mathsf{base}(\mathcal{B})$ we conclude that $\mathsf{base}(\mathcal{B}) \supsetneq \mathsf{base}(\mathcal{A})$, so $\mathcal{B}$ is a (proper) defeater for $\mathcal{A}$. Thus, $[\mathcal{A}, \mathcal{B}, \ldots]$ is in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{A})$. From this and the assumption that $\mathcal{B}$ is undefeated in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{B})$, we can apply Lemma 1 to show that $\mathcal{B}$ is undefeated in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{A})$. Hence, $\mathcal{A}$ is defeated in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{A})$ (contradiction). (Case) If $\mathcal{A}$ is a defeater for $\mathcal{B}$, then $[\mathcal{B}, \mathcal{A}, \ldots]$ is in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{B})$, so by assumption on $\mathcal{B}$, $\mathcal{A}$ is defeated in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{B})$. Then, by Lemma 1, we obtain that $\mathcal{A}$ is defeated in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{A})$ (contradiction). Hence $\langle \sim\ell, t \rangle \notin \mathsf{warr}(\Pi, \Delta)$. Since $\ell$ and $t$ were arbitrary, $\mathsf{warr}(\Pi, \Delta)$ is consistent. □

## 4 Nature does not wait: eager arguments.

We study in this section a sub-class of $t$-DeLP arguments, called *eager*, for reasoning with natural processes. In law-governed processes *as soon as* all conditions hold, the

[6] Recall that, according to Defintion 1, $\mathsf{warr}(\Pi, \Delta)$ is consistent iff there is no $p$ such that both $p$ and $\sim p$ belong to $\mathsf{warr}(\Pi, \Delta)$. This differs from stronger notions of consistency requiring that $\mathsf{warr}(\Pi, \Delta) \cup \Pi$ does not derive any pair of contradictory literals.

process can do nothing else than start. This would exclude from the class of arguments that model some natural process those constructible arguments that unnecessarily postpone the (start of an) application of a rule after its body holds (i.e. arguments that introduce some unnecessary delay after the rule becomes *applicable*). No natural process corresponds to these $t$-DeLP arguments, so they should be excluded from reasoning about natural processes.

Interestingly, any argument $\mathcal{A}$ can be transformed into an eager argument $\mathcal{A}^*$ by following an iterative procedure. The idea is that $\mathcal{A}^*$ orders non-persistence (resp. persistence) rules in $\mathcal{A}$ to occur as early (resp. late) as possible while keeping the same base. To obtain $\mathcal{A}^*$, let initially $\mathcal{A}' = \mathcal{A}$, and apply iteratively the following transformation on $\mathcal{A}'$ until it cannot be applied any longer:

1. Select a rule $\delta^* \in \mathcal{A}'$ such that the next condition holds: for each $\langle \ell_i, t_i \rangle \in \mathsf{body}(\delta^*)$, there exists (at least) an instance of the persistence rule $\delta_{\ell_i}$ supporting this $\langle \ell_i, t_i \rangle$. If there is no such a rule let $\mathcal{A}^* = \mathcal{A}'$ and STOP, otherwise follow to the next step
2. For each $\langle \ell_i, t_i \rangle \in \mathsf{body}(\delta^*)$, let $\{\langle \ell_i, t_i \rangle \prec \langle \ell_i, t_i - 1 \rangle, \ldots, \langle \ell_i, t_i - k_i + 1 \rangle \prec \langle \ell_i, t_i - k_i \rangle\} \subseteq \mathcal{A}_n$ be the set of persistence rules for $\ell_i$ in $\mathcal{A}'$ such that $\langle \ell_i, t_i - k_i \rangle \in \mathsf{literals}(\mathcal{A}_n)$ is supported by some non-persistence rule. Let $k_j$ be such that the difference $t_j - k_j$ is minimal among those related to $\mathsf{body}(\delta^*)$.
3. Define a new rule, denoted $\delta^* - k_j$, as the rule where the temporal parameter of each literal $\langle \ell, t \rangle \in \mathsf{literals}(\delta^*)$ is subtracted $k_j$. Then, to obtain $\mathcal{A}_{n+1}$, we:
4. In $\mathcal{A}'$ replace $\delta^*$ by $\delta^* - k_j$
5. For each $\langle \ell_i, \cdot \rangle \in \mathsf{body}(\delta^*)$, delete from $\mathcal{A}'$ the $t_j - k_j$ instances of persistence rules $\delta_{\ell_i}$ of the form:
$$\langle \ell_i, t_j \rangle \prec \langle \ell_i, t_j - 1 \rangle, \ldots, \langle \ell_i, t_j - k_j + 1 \rangle \prec \langle \ell_i, t_j - k_j \rangle$$
6. If $\mathsf{head}(\delta^*) = \langle \ell, t \rangle$, add to $\mathcal{A}'$ the $t_j - k_j$ instances of persistence rule $\delta_\ell$ of the form: $\langle \ell, t \rangle \prec \langle \ell, t-1 \rangle, \ldots \langle \ell, t - (t_j - k_j) + 1 \rangle \prec \langle \ell, t - (t_j - k_j) \rangle$.
7. Let $\mathcal{A}'$ be the new argument resulting after the above steps. Start the procedure again.

The ouput of the above procedure, $\mathcal{A}^*$, is an argument sharing many properties with $\mathcal{A}$: $\mathsf{base}(\mathcal{A}^*) = \mathsf{base}(\mathcal{A})$, $\mathsf{concl}(\mathcal{A}^*) = \mathsf{concl}(\mathcal{A})$, hence $\|\mathcal{A}^*\| = \|\mathcal{A}\|$ and $\mathbf{t}(\mathcal{A}^*) = \mathbf{t}(\mathcal{A})$. See Figure 4 for an instance of this transformation, where the leftmost argument is an eager transformation of the argument at the center. It can be observed that these transformations define an equivalence relation $\equiv_{\mathsf{p}}$ on the set of arguments in $(\Pi, \Delta)$: $\mathcal{B}, \mathcal{C}$ are equivalent iff $\mathcal{B}^* = \mathcal{C}^*$.

*Example 4.* For an intuitive counterexample to the defeat relation in Definition 7, consider the arguments represented in Figure 4. The rightmost argument $\mathcal{A}$ is lengthier than $\mathcal{B}^*$, hence a proper defeater for the latter. The argument at the center, $\mathcal{B}$, is more suspicious than $\mathcal{B}^*$ since it assumes that the latter holds even if when applying persistence to its base. Note that $\mathcal{B}^*$ is eager and equivalent to $\mathcal{B}$. The problem with the definition of proper defeater, is that $\mathcal{A}$ is not lengthier than $\mathcal{B}$, hence it can only be a blocking defeater for $\mathcal{B}$. But intuitively, $\mathcal{A}$ should be a proper defeater for $\mathcal{B}$ as well.

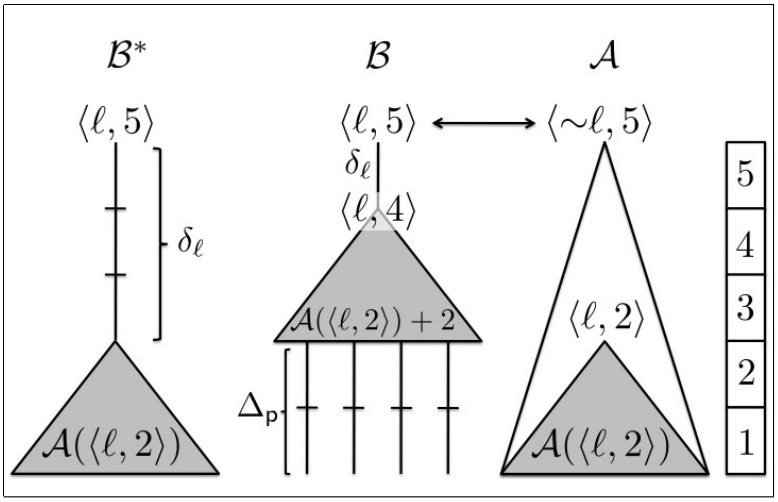


**Fig. 4.** The argument $\mathcal{A}$ should be a proper defeater for $\mathcal{B}$, as it is for $\mathcal{B}^*$

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Counterexamples like those of Figure 4 can be pruned either by

(a) restricting to the set of eager arguments, i.e. to the set of arguments obtaining the previous procedure applied to all the arguments of a given $t$-de.l.p. $Args(\Pi, \Delta) \mapsto Args^*(\Pi, \Delta)$, or
(b) by redefining the notion of *lengthier argument* so that $\mathcal{A}$ is a lengthier argument than $\mathcal{B}$ iff $\mathcal{A}$ is lengthier than $\mathcal{B}^*$ (in the old sense of Definition 7), where $\mathcal{B}^*$ is the eager argument in $[\mathcal{B}]_{\equiv_{\mathsf{p}}}$.

In the following we prove the two options are equivalent. From now on, all the definitions (from proper defeater to $\mathsf{warr}(\cdot,\cdot)$) are assumed as including the modification on the definition for *lengthier argument* given by (b).

**Proposition 1.** *Let $(\Pi, \Delta)$ be a t-de.l.p. and let $\mathcal{A}_0$ be an eager argument in $(\Pi, \Delta)$. Then, for any other arguments $\mathcal{A} \in [\mathcal{A}_0]_{\equiv_{\mathsf{p}}}$ and $\mathcal{B}$ in $(\Pi, \Delta)$ we have:*

*(1) if $\mathcal{B}$ is a proper (blocking) defeater for $\mathcal{A}_0$ then so is $\mathcal{B}$ for $\mathcal{A}$.*
*(2) if $\mathcal{A}$ is a proper (blocking) defeater for $\mathcal{B}$, so is $\mathcal{A}_0$.*

Claim (1) shows eager arguments are the safest among their $\equiv_{\mathsf{p}}$ equivalence class in $(\Pi, \Delta)$. Define $\mathcal{T}^*_{(\Pi,\Delta)}$ and $\mathsf{warr}^*(\Pi, \Delta)$ as the dialectical tree and warranted set in the restriction to the class of eager arguments $Args^*(\Pi, \Delta)$. e.g. $\ell \in \mathsf{warr}^*(\Pi, \Delta)$ iff there exists an eager argument $\mathcal{A}_0$ in $(\Pi, \Delta)$ undefeated in $\mathcal{T}^*_{(\Pi,\Delta)}(\mathcal{A}_0)$. (These two definitions $\mathcal{T}^*_{(\cdot,\cdot)}(\cdot)$ and $\mathsf{warr}^*(\cdot,\cdot)$ are the original ones, without the modification suggested in point (b) above.)

**Corollary 1.** *Fix a t-de.l.p. $(\Pi, \Delta)$. Let $\mathcal{A}_0$ be an eager argument. Then $\mathcal{A}_0$ is undefeated in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{A}_0)$ iff it is undefeated in $\mathcal{T}^*_{(\Pi,\Delta)}(\mathcal{A}_0)$. As a consequence,* $\mathsf{warr}(\Pi, \Delta) = \mathsf{warr}^*(\Pi, \Delta)$.

## Conclusions and Future Work

We have presented $t$-DeLP a temporal extension of DeLP with temporal literals and rules with duration. Indeed one can think of the DeLP framework to correspond to $t$-DeLP (with strict rules made explicit in arguments and with only one time-point, e.g. when all temporal literals are of form $\langle \ell, 0 \rangle$). Other rule-based defeasible logics[2], [8] exist as well and have a considerable literature, but they differ from our approach in the modularity and conceptual transparency of the logical machinery. The same goes for defeasible logics extended with temporal parameters associated to literals [6]. On the other hand, argumentation-based logical approaches (inspired by the work of [4]) do not in general take the particularities involved in temporal reasoning into account. Among works that do consider argumentation and time, we find some proposals associating time intervals to arguments [1], [3] and [7]. Our approach differs from these works in that the *interval where an event or argument holds*, rather than being a primitive notion, derives from the argumentation process. Thus, our time-point based approach accommodates features from [6], [1] (expiring literals, persistence), though in the present paper these features are subject to argumentation processes, rather than having them fixed from start.

For future work, we would like to expand temporal reasoning with evidence-based reasoning from future to the past, among many other improvements on the generality of our language or the results we obtained.

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