A note on the hierarchy of algebraizable logics

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A logic L (a structural consequence relation) in a language $\mathcal{L}$ is *algebraizable* [1, 3] w.r.t. a class of L-algebras $\mathbb{L}$ with translations $\rho\colon \mathrm{Eq}_{\mathcal{L}} \to \mathcal{P}(\mathrm{Fm}_{\mathcal{L}})$ and $\tau\colon \mathrm{Fm}_{\mathcal{L}} \to \mathcal{P}(\mathrm{Eq}_{\mathcal{L}})$[1] if

1. $\Pi \models_{\mathbb{L}} \varphi \approx \psi$ iff $\rho[\Pi] \vdash_{\mathrm{L}} \rho(\varphi \approx \psi)$
2. $p \dashv\vdash_{\mathrm{L}} \rho[\tau(p)]$

There are numerous strengthenings of this notion in the literature, which are often confused, the usually mistakes being that finitary of L implies that $\mathbb{L}$ is an elementary class[2] (a counterexample is given in [2]) or vice versa (a counterexample is given in [4]). Moreover, the relation of these two notions with the finiteness of ρ (called *finite algebraizability*) is another usual source of confusions.

The goal of this talk is to clarify these confusions by considering the overlooked condition of finiteness of τ. We show that by combining these four properties we obtain 7 distinct classes of logics (the smallest class coinciding with that of *B–P algebraizable logics* [1]). Then we add two more well-studied properties: regularity of algebraization (a special requirement for τ) and algebraic implicativeness of L (a special requirement for ρ). We eventually obtain a hierarchy of 17 classes logics of algebraizable logics and show their separation examples.

References

[1] Willem J. Blok and Don L. Pigozzi. *Algebraizable Logics*. Memoirs of the American Mathematical Society 396, 1989.

[2] Pilar Dellunde. A Finitary 1-Equivalential Logic Not Finitely Equivalential. *Bulletin of the Section of Logic* 24:120–122, 1995.

[3] Janusz Czelakowski. *Protoalgebraic Logics*, volume 10 of *Trends in Logic*. Kluwer, 2001.

[4] James G. Raftery. A Non-finitary Sentential Logic that is Elementarily Algebraizable. *Journal of Logic and Computation* 20(4):969–975, 2010.

[1] We set $\rho[\Pi] = \bigcup_{\psi\in\Pi} \rho(\psi)$ and analogously for τ.
[2] Actually a quasivariety.