# Probabilistic argumentation: an approach based on conditional probability – a preliminary report–

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**Abstract.** A basic form of an instantiated argument is as a pair (support, conclusion) standing for a conditional relation 'if support then conclusion'. When this relation is not fully conclusive, a natural choice is to model the argument strength with the conditional probability of the conclusion given the support. In this paper, using a very simple language with conditionals, we explore a framework for probabilistic logic-based argumentation based on an extensive use of conditional probability, where uncertain and possibly inconsistent domain knowledge about a given scenario is represented as a set of defeasible rules quantified with conditional probabilities. We then discuss corresponding notions of attack and defeat relations between arguments, providing a basis for appropriate acceptability semantics, e.g. based on extensions or on DeLP-style dialogical trees.

## 1 Introduction

In the literature, there have been a number of approaches [5,15,25,10,11,12,21,13,19,3] to combine different theories of argumentation with probability theory, and other uncertainty models, in order to allow for a more fine-grained reasoning when arguments involve uncertain information. Since the earliest works of Pollock [17,18], where he introduced the notion of strength of an argument in terms of numerical degrees of belief, one main open problem has been to determine how the strength of arguments can be related to probability theory, see e.g. [19].

In [23], arguments are generated in ASPIC+ and their rebutting attacks are resolved with probabilistic strengths of arguments. However, some difficulties are encountered when assigning probabilities to arguments in an abstract framework. In a natural way, probabilities can be assigned to the truth of statements or to outcomes of events, but an argument is neither a statement nor an event. Thus, there is a need for a meaningful definition of what the probability of an argument is, and this has to be done at the level of structured argumentation, for instance along the line of the epistemic approach to probabilistic argumentation [10,20,19]. In particular, in the setting of classical-logic based argumentation, Hunter considers in [10] the probability of an argument to be the probability of its premises according to a fixed, and a priori given, probability distribution on the set of interpretations of the language. Similarly, In [19], Prakken discusses the application of the ASPIC+ framework to default reasoning with probabilistic generalisations, taking the probability of an argument to be the probability of the conjunction of all its premises and conclusions.

In contrast to [10] but similarly to [19], in this paper we consider logic-based arguments $A = (support; conclusion)$ pervaded with uncertainty due a non-conclusive conditional link between their supports and their conclusions. In such a case, it is very reasonable to supplement the argument representation with a quantification $\alpha$ of how certain *conclusion* can be claimed to hold whenever *support* is known to hold, leading to represent an arguments as triples $A = (support; conclusion : \alpha)$. A very natural choice is to interpret $\alpha$ as a conditional probability, namely the probability $P(conclusion \mid support)$. As we frame our proposal in logic-based argumentation, where arguments rely on the notion of proof in some underlying logic, we internalise the conditional link specified by an argument in the logic as a conditional formula or a set of conditional formulas in the general case, so that our basic probabilistic arguments will be of the form

$$A = (\{\psi\}, \{\psi \rightsquigarrow \varphi : \alpha\}; \varphi : \alpha),$$

where $\psi$ and $\varphi$ are classical propositions, $\psi \rightsquigarrow \varphi$ is a conditional formula and $\alpha$ is interpreted as a lower bound for the conditional probability $P(\varphi \mid \psi)$. When arguments get more complex and need several uncertain conditionals to link the support with the conclusion, conditional probabilities are attached to each of the involved conditionals, so arguments become of the form

$$A = (\Pi, \Delta = \{(\psi_1 \rightsquigarrow \varphi_1 : p_1), \ldots, (\psi_n \rightsquigarrow \varphi_n : p_n)\}; \varphi : \alpha),$$

where $\Pi$ is a finite set of factual (i.e. non conditional) premises and $\alpha$ the probability with which $\varphi$ can be logically entailed from $\Pi$ and $\Delta$. In fact, this type of arguments can be seen as a probabilistic generalization of those at work in the Defeasible Logic Programming argumentation framework (DeLP) [7]. This is a formalism combining logic programming and defeasible argumentation, that provides the possibility of representing information in the form of weak rules and a defeasible argumentation inference mechanism for warranting the entailed conclusions, see [8] for more details.

Our proposal can be cast in the above mentioned epistemic approach that assigns probabilities to arguments. However, in contrast to many works in the literature, we do not assign probabilities to the arguments a priori, but rather use smaller pieces of probabilistic information that govern the universe of study, and use these to compute the probability of a complex argument built from the more basic information items it contains. Moreover, our approach also notably differs from previous schemes in that, to compute the probability for an argument, we consider the whole family of probability distributions compatible with the support, and not fixing only one distribution

This paper is structured as follows. Section 2 is devoted to introduce notions about logic and probability necessary for the rest of the paper; in Section 3 we introduce and explore the framework of probabilistic argumentation based on conditional probabilities. We conclude the paper commenting on promising future work and open questions.

## 2 Logic and probability

When aiming towards the definition of a formal argumentation framework, a first step is the selection of a underlying purely propositional language and the logical system that will govern the derivation of new knowledge from a given set of information. In this paper, our logical formalism will be inspired in DeLP [7].

Let $\mathcal{V}$ be the set of ***propositional variables***, simply a countable set of symbols. A ***literal*** is any propositional variable $x \in \mathcal{V}$ or a negated variable $\neg x$ for $x \in \mathcal{V}$. If $\ell$ is a literal, we will use the notation $\neg\ell$ to refer to $x$ if $\ell = \neg x$ and to $\neg x$ if $\ell = x$. A ***conjunction*** of literals is a formula of the form $\ell_1 \wedge \ldots \wedge \ell_n$ with $n \geq 1$, where each $\ell_i$ is a literal. A ***conditional*** is a formula of the form $\ell_1 \wedge \ldots \wedge \ell_n \rightsquigarrow \ell$. Finally, we call ***formula*** any conjunction or conditional, and denote the set of formulas by $Fm$. Given a set of formulas $\Psi \subseteq Fm$, we will denote by $lit(\Psi)$ the set of literals appearing in $\Psi$.

**Definition 1 (c.f. Def. 2.5 from [7]).** *Let $\Sigma$ be a finite set of conditionals, $\Phi$ a finite set of literals and $\ell$ a literal. A DeLP derivation of $\ell$ from $\Sigma$ and $\Phi$, denoted $\Sigma, \Phi \vdash \ell$, is a finite sequence $\ell_1, \ldots, \ell_n = \ell$ of literals, such that, for each $1 \leq i \leq n$:*
- *a) either $\ell_i \in \Phi$, or*
- *b) there is a conditional $p_1 \wedge \ldots \wedge p_k \rightsquigarrow p \in \Sigma$ such that $p = \ell_i$ and for each $1 \leq j \leq k$, $p_j \in \{l_1, \ldots \ell_{i-1}\}$.*

A pair $\{\Sigma, \Phi\}$ is *consistent* if it is not the case that there exists a literal $\ell$ such that both $\Sigma, \Phi \vdash \ell$ and $\Sigma, \Phi \vdash \neg\ell$.

Let $\Omega$ stand for the set of truth-evaluations of variables $e : \mathcal{V} \to \{0, 1\}$, that extend to literals and conjunctions of literals following the rules of classical logic. Probabilities on the set of formulas $Fm$ are defined in the standard way, as it is done in probability logics: defining a probability distribution on $\Omega$ and extending it to all formulas by adding up the probabilities of their models. More precisely, let $P\colon \Omega \to [0, 1]$ be a probability distribution on $\Omega$. Then $P$ induces a probability[3] $P\colon Fm \to [0, 1]$ by letting:

- $P(C) = \Sigma_{e\in\Omega, e(C)=1} P(e)$, if $C$ is a conjunction of literals,
- $P(\ell_1 \wedge \ldots \wedge \ell_n \rightsquigarrow \ell) = P(\ell \wedge \ell_1 \wedge \ldots \wedge \ell_n)/P(\ell_1 \wedge \ldots \wedge \ell_n)$, whenever $P(\ell_1 \wedge \ldots \wedge \ell_n) > 0$ and undefined otherwise. Namely, the probability of $\ell$ conditioned to $\ell_1 \wedge \ldots \wedge \ell_n$.

Notice that the probability of a conditional $C \rightsquigarrow \ell$ is interpreted as the conditional probability $P(\ell \mid C)$, not as a probability of the material implication $\neg C \vee \ell$, understood as the implication in classical logic. Nevertheless, these two notions do coincide when the probability equals to 1. Namely, for $P(C) > 0$ for a conjunction of literals $C$, then

$$P(C \rightsquigarrow \ell) = 1 \text{ if and only if } P(\neg C \vee \ell) = 1.$$

We will call ***probabilistic-valued formulas*** (and denote this set of formulas by $Fm_{Pr}$) to all pairs of the form $\varphi\colon \alpha$, where $\varphi \in Fm$ and $\alpha \in [0, 1]$. A probability $P\colon \Omega \to [0, 1]$ satisfies $\varphi\colon \alpha$, written $P \models \varphi\colon \alpha$, whenever $P(\varphi) \geq \alpha$. Similarly, $P$ satisfies a finite set of valued formulas $\Sigma = \{\varphi_i\colon \alpha_i\}_{i\in I}$ if it satisfies each pair in $\Sigma$. We will denote the set of probabilities that satisfy $\Sigma$ by $PMod(\Sigma)$.

[3] Since there is no place to confusion, we will use the same symbol $P$ to denote the probability distribution over $\Omega$ and its associated probability over $Fm$.

Given a set of literals $\Pi$ representing observations on the domain, one can define two probabilistic consequence relations, depending on how the set of observations $\Pi$ is interpreted: either as facts holding with probability 1, or as assumptions over which to condition the consequence. These two definitions are intrinsically related to the two types of arguments we will introduce in the next section.

**Definition 2 (Factual probabilistic entailment).** *Let $\Pi$ be a set of literals, $\Sigma$ a set of valued formulas, $\ell$ a literal and $\alpha \in [0,1]$. We write $\Pi, \Sigma \models^{f}_{Pr} \ell : \alpha$ whenever for each probability $P \in PMod(\Sigma)$, if $P(c) = 1$ for each $c \in \Pi$ then $P(\ell) \geq \alpha$.*

**Definition 3 (Conditioned probabilistic entailment).** *Let $\Pi$ be a set of literals, $\Sigma$ a set of valued formulas, $\ell$ a literal and $\alpha \in [0,1]$. We write $\Pi, \Sigma \models^{c}_{Pr} \ell : \alpha$ whenever for each probability $P \in PMod(\Sigma)$, it holds that $P(\bigwedge_{c \in \Pi} c \rightsquigarrow \ell) \geq \alpha$.*

These two notions of entailment do not coincide. First observe that the conditioned probabilistic entailment is **stronger** than the unconditioned one, namely $\Pi, \Sigma \models^{c}_{Pr} \ell : \alpha$ implies $\Pi, \Sigma \models^{f}_{Pr} \ell : \alpha$. However, the converse does not hold, i.e. the conditioned probabilistic entailment is **strictly stronger** than the factual one. For instance, if we take the observation $\Pi = \{a\}$ and the valued formulas $\Sigma = \{a \rightsquigarrow b : 0.7, b \rightsquigarrow c : 0.5\}$, it t is easy to check that $\Pi, \Sigma \models^{f}_{Pr} c : 7/20$, but $\Pi, \Sigma \not\models^{c}_{Pr} c : 7/20$.

## 3 Using conditional probability in arguments

Our approach is inspired DeLP, ASPIC+ and other systems that differentiates knowledge that is certain and consistent (strict) from other that is tentative and possibly uncertain and inconsistent (defeasible). Probabilities offer a finer classification of the uncertain knowledge and so increase the trustworthiness and accurateness of arguments. In this paper, we assume the strict domain knowledge to come attached with probability 1, but other values could be used (e.g. if precise statistical data is possessed).

**Definition 4.** *$\mathcal{K} = \langle \Pi, \Delta \rangle$ is a probabilistic conditional knowledge base (KB) whenever*

- *$\Pi = \Pi_F \cup \Pi_D \subseteq Fm$ is a consistent[4] set of formulas encompassing the strict knowledge in $\mathcal{K}$, divided in factual knowledge ($\Pi_F$) under the form of literals, and domain knowledge ($\Pi_D$) under the form of strict rules.*
- *$\Delta \subseteq Fm_{Pr}$ encompasses uncertain probabilistic knowledge.*

*Example 1.* The following KB is a probabilistic refinement of Example 2.1 in [7], a variant of the famous Tweety example. Chickens usually do not fly (even if they are birds), but they may if they are scared, for instance if a fox is near. However, if a chicken has nestling babies, most likely it will not abandon them in any case.

$$\Pi_F = \begin{Bmatrix} chicken \\ fox \\ nestlings \end{Bmatrix} \qquad \Delta = \begin{Bmatrix} bird \rightsquigarrow flies : 0.85 \\ chicken \rightsquigarrow \neg flies : 0.9 \\ chicken \wedge nestlings \rightsquigarrow \neg flies : 0.95 \\ chicken \wedge fox \rightsquigarrow scared : 0.8 \\ chicken \wedge scared \rightsquigarrow flies : 0.6 \end{Bmatrix}$$

$$\Pi_D = \{ chicken \rightsquigarrow bird \}$$

[4] According to $\vdash$.

To specify an argument, we needed to specify which observations and which (consistent) part of the uncertain probabilistic knowledge it is based upon. We propose two main definitions for a probabilistic argument, each one following relying in one of the definitions of probabilistic entailment from the previous section. In what follows, for a set of formulas $\Gamma \subseteq Fm$ we let $\Gamma^+ = \{\gamma\colon 1\}_{\gamma\in\Gamma} \subseteq Fm_{Pr}$. Conversely, for a set of valued formulas $\Sigma \subseteq Fm_{Pr}$, we let $\Sigma^- = \{\sigma \mid \sigma\colon \alpha \in \Sigma \text{ for some } \alpha \in [0,1]\} \subseteq Fm$.

**Definition 5 (Argument).** *Let $\star \in \{f, c\}$[5], and a $KB = \langle \Pi, \Delta\rangle$. A $\star$-probabilistic argument $\mathcal{A}$ for a literal $\ell$ in KB is a structure $A = \langle \Phi, \Gamma; \ell\colon \alpha\rangle$, where $\Phi \subseteq \Pi_F$, $\Gamma = \{(\varphi_1 \rightsquigarrow l_1\colon \alpha_1), \ldots, (\varphi_n \rightsquigarrow l_n\colon \alpha_n)\} \subseteq \Delta$ and $\alpha > 0$ such that:*

(1) $PMod(\Gamma \cup \Pi^+) \neq \emptyset$

(2) $\Pi, \Gamma^- \vdash \ell$

(3) $\alpha = \max\{\, \beta \in [0,1] : \Phi, \Pi_D^+ \cup \Gamma \models^{\star}_{Pr} \ell\colon \beta \,\}$

(4) *$\Phi$ and $\Gamma$ are minimal satisfying* (1), (2) *and* (3).

Thus, an argument for a literal provides for both a logical and an optimal probabilistic derivation of its conclusion (in any of the two variants) from its premises.

Some simple examples of probabilistic arguments over the KB from Example 1 are:

$$\mathcal{A}_1 = (\{chicken\}, \{bird \rightsquigarrow flies\colon 0.85\}; flies\colon 0.85)$$
$$\mathcal{A}_2 = (\{chicken\}, \{chicken \rightsquigarrow \neg flies\colon 0.9\}; \neg flies\colon 0.9)$$
$$\mathcal{A}_3 = (\{chicken, fox\}, \{chicken \wedge fox \rightsquigarrow scared\colon 0.8, chicken \wedge scared \rightsquigarrow flies\colon 0.6\}; flies\colon 0.54)$$
$$\mathcal{A}_4 = (\{chicken, nestlings\}, \{chicken \wedge nestlings \rightsquigarrow \neg flies\colon 0.95\}; \neg flies\colon 0.95)$$

$\mathcal{A}_1$, $\mathcal{A}_2$ and $\mathcal{A}_4$ are both $f$- and $c$-arguments, while $\mathcal{A}_3$ is a $f$-argument but not a $c$-argument. This occurs because $\models^{c}_{Pr}$ becomes non-informative (its degree equals 0) when its logical derivation involves the chaining of more than one conditional, due to the well-known failure of transitivity on conditional probabilities [9], unless some additional assumptions are made. For instance, in [19] arguments implicitly make probabilistic independence assumptions and it is shown that the independence assumptions, that justify the use a version of the chain rule for probabilities, is useful in certain cases, but it is clearly invalid in general.

In order to define an attack relation between probabilistic arguments, we need the notions of subargument and of disagreement between probabilistic-valued literals.

**Definition 6 (Subargument, Disagreement and Attack).** *1) Let $\mathcal{A} = (\Phi, \Gamma; \ell\colon \alpha)$ be an $\star$-argument for $\ell$. A subargument of $\mathcal{A}$ is an $\star$-argument $\mathcal{B} = (\Phi', \Gamma'; \ell'\colon \beta)$ where $\Phi' \subseteq \Phi$ and $\Gamma' \subseteq \Gamma$.*

*2) Let $KB = (\Pi, \Delta)$ be a knowledge base. We say that the valued-literals $\ell : \alpha$ and $h : \beta$ disagree whenever they are probabilistically inconsistent with the strict knowledge, i.e. when $PMod(\Pi^+ \cup \{l{:}\alpha, h{:}\beta\}) = \emptyset$.*

*3) A $\star$-argument $\mathcal{A} = (\Phi_1, \Gamma_1; \ell, \alpha)$ attacks another $\star$-argument $\mathcal{B} = (\Phi_2, \Gamma_2; p\colon \beta)$ at a literal $h$ if there is a $\star$-subargument $\mathcal{B}' = (\Phi_2', \Gamma_2'; h\colon \gamma)$ of $\mathcal{B}$ such that $\ell : \alpha$ and $h : \gamma$ disagree.*

[5] Standing for factual or conditioned arguments.

Using only the probabilities to determine when an attack can be deemed as effective may be counterintuitive in some cases (see e.g. arguments $\mathcal{A}_2$ and $\mathcal{A}_3$), thus we combine them with the use of specificity criterion (gaining inspiration in [7,1,2])

**Definition 7 (Activation sets and Specificity).** *Given a knowledge base KB, an* activation set *of an argument* $\mathcal{A} = (\Phi, \Gamma; \ell, \alpha)$ *is a set of literals* $H \subseteq lit(KB)$ *such that* $H \cup \Pi_D \cup \Gamma^- \vdash \ell$. *We denote by* $Act(\mathcal{A})$ *the set of activation sets for the argument* $\mathcal{A}$.

*An argument* $\mathcal{A}$ *is* more specific than *another argument* $\mathcal{B}$ *when* $Act(\mathcal{A}) \subsetneq Act(\mathcal{B})$. $\mathcal{A}$ *and* $\mathcal{B}$ *are equi-specific if* $Act(\mathcal{A}) = Act(\mathcal{B})$, *and incomparable whenever* $Act(\mathcal{A}) \nsubseteq Act(\mathcal{B})$ *and* $Act(\mathcal{A}) \nsupseteq Act(\mathcal{B})$.

In our running example, we can easily check that $\mathcal{A}_3$ and $\mathcal{A}_4$ are incomparable, and both are more specific than $\mathcal{A}_2$, which is itself more specific than $\mathcal{A}_1$.

**Definition 8 (Strength and Defeat).** *An argument* $\mathcal{A} = (\Phi_1, \Gamma_1; \ell\colon \alpha)$ *is stronger than another argument* $\mathcal{B} = (\Phi_2, \Gamma_2; p\colon \beta)$ *when* $\mathcal{A}$ *is more specific than* $\mathcal{B}$, *or when* $\mathcal{A}$ *and* $\mathcal{B}$ *are equi-specific or incomparable and* $\alpha > \beta$.

*An argument* $\mathcal{A} = (\Phi_1, \Gamma_1; \ell\colon \alpha)$ *defeats another argument* $\mathcal{B} = (\Phi_2, \Gamma_2; p\colon \beta)$ *when* $\mathcal{A}$ *attacks* $\mathcal{B}$ *on a subargument* $\mathcal{B}' = (\Phi_2', \Gamma_2'; h\colon \gamma)$ *and* $\mathcal{A}$ *is stronger than* $\mathcal{B}'$.

Following with the running example, we have that $\mathcal{A}_2$ defeats $\mathcal{A}_1$, and $\mathcal{A}_3$ defeats $\mathcal{A}_2$ based on the specificity criterion. On the other hand $\mathcal{A}_4$ defeats $\mathcal{A}_3$ on the basis of probability degree criterion, while it defeats $\mathcal{A}_2$ due to specificity.

The proposed setting serves to define an argumentation semantics by considering an argumentation theory and substituting the notions of argument, attack and defeat from the original theory by the ones we propose here. In this fashion, it is natural how to produce argumentation systems with different high-level semantics: from Dung's abstract argumentation systems [4], or other relevant weighted argumentation systems based on it (eg. [10]), to the rule-based DeLP argumentation framework and its dialectical-tree based semantics [7], or other systems like ASPIC$^+$ [16] or ABA [24]. The definition of the systems is rather immediate and we do not detail them here due to a lack of space. However, the exploration of the resulting systems and the differences with the original ones will involve more work, and we leave it for future work.

## 4 Future work

Plenty of issues could be worked out and studied in future works. First, it seems likely that in certain situations, a richer language of conditionals would be useful, eg. considering conditional logics in the style of Kern-Isberner's three-valued conditionals [14] or the logic of Boolean conditionals [6]. Secondly, other interpretations of the probability entailment can be explored: for instance, to allow for interpreting the weights in valued formulas not only as a lower bound but with other constraints like an equality or a strict lower bound, or to compute the probability of the conclusion of an argument by means of the Maximum Entropy distribution underlying the premises [26,22]. Lastly, a finer gradual notion of attack could be introduced so to allow an attacker argument to debilitate the attacked argument, instead of an all-or-nothing attack.

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