N is a fuzzy negation with a unique point of discontinuity are presented. The proofs are twofold. First, applying similar arguments to the case of the characterization of (S, N)-implications derived from continuous negations, the underlying continuous t-conorm S is defined from the values of the fuzzy implication function, but not in the whole unit square. Therefore, this partial t-conorm must be completed to the whole unit square. This completion is performed by means of similar techniques to the ones used in [3].

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References:

[1] M. Baczynski, B. Jayaram, S. Massanet, J. Torrens, Fuzzy Implications: Past, Present, and Future, in J. Kacprzyk and W. Pedrycz (eds.), Springer Handbook of Computational Intelligence, Springer Berlin Heidelberg, pp. 183–202 (2015).

[2] M. Baczynski, B. Jayaram: On the characterizations of (S,N)-implications. Fuzzy Sets and Systems 158(15): 1713-1727 (2007).

[3] A. Mesiarová-Zemánková: Continuous completions of triangular norms known on a subregion of the unit interval. Fuzzy Sets and Systems 308: 27-41 (2017).

Pattern Classification on Complex System Using Modified Gustafson-Kessel Algorithm

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This work has been focused in the application of Gustafson-Kessel Algorithm in a complex system through a methodology proposed. The complex system here considered will be the financial market. So, the main objective of this paper is to classify objects in two patterns: winner and loser. The methodology is based on application of a method of clustering called Modified Gustafson-Kessel (MGK) in some open companies of the transportation sector and energy sector. Results shows that the use of MGK can better separate the promising actions from the non-promising ones with more precision due to its covariance matrix that can be change for generate the best separability among clusters. This produces a new tool for analysis of the dynamic of stock market with the main aim of given support to investor in make decision.

A Representation Theorem for Finite Gödel Algebras with Operators

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Modal extensions of fuzzy logics is an active and relatively recent area of research. In a nutshell, the main aim of this approach is to generalize the semantics of classical modal logic by allowing the evaluation in each world and the accessibility relation to take values in the real unit interval $[0, 1]$, instead of the classical two-valued set $\{0, 1\}$ (see [3,4] for instance).

In this contribution we will present an algebraic-oriented approach to modal fuzzy logic focusing on Gödel modal logic by defining and studying finite Gödel algebras with operators (GAOs for short). Those are triples $(A, \Diamond, \Box)$ where A is a finite Gödel algebra [3], $\Diamond$ and $\Box$ are unary modal operators on A satisfying the following equations:

$$\Diamond\bot = \bot; \Diamond(a \vee b) = \Diamond a \vee \Diamond b;$$
$$\Box T = T; \Box(a \wedge b) = \Box a \wedge \Box b.$$

Notice that the equations which describe the behavior of modal operators in a GAO are the same which characterize $\Diamond$ and $\Box$ in a boolean algebra with operators (BAO) (see [2]). Obviously, while in a BAO $\Diamond$ and $\Box$ are inter-definable, Gödel negation, being not involutive, does not allow such a simplification and hence the equation $\Diamond a = \neg\Box\neg a$ does not hold in general in a GAO.

In the same way the dual frames of boolean algebras with operators are Kripke frames, taking into account the duality between finite Gödel algebras and finite forests [1], we introduce dual structures of Gödel algebras with operators as triples (F, D, B) called forest-frames, where $F = (F, \leq)$ is a finite forest and D and B are binary relations on F satisfying the following conditions: for all $x, y, z \in F$

(A) if $x \leq y$, then $D(x, z) \geq D(y, z)$
(anti-monotonicity in the first argument);

(M) if $x \leq y$, then $B(x, z) \leq B(y, z)$
(monotonicity in the first argument).

If $(A, \Diamond, \Box)$ is a GAO and $F(A)$ denotes the finite forest of its prime filters, then there exists a standard way to define binary relations $R(\Diamond)$ and $R(\Box)$ on $F(A)$ in such a way that $(F(A), R(\Diamond), R(\Box))$ is a forest frame. Vice-versa, given a forest-frame (F, D, B), letting $G(F)$ be the Gödel algebra of subforests of F, there is also a canonical way to define operators $O(D)$ and $O(B)$ on $G(F)$ such that $(G(F), O(D), O(B))$ is a GAO.

Our main result is a Jónsson-Tarski like representation theorem for finite GAOs:

Theorem 1. For every finite GAO $(A, \Diamond, \Box)$, let $(F(A), R(\Diamond), R(\Box))$ be as above. Then $(A, \Diamond, \Box)$ and $(G(F(A)), O(R(\Diamond)), O(R(\Box)))$ are isomorphic Gödel algebras with operators.

Further, we will discuss the effect of a stronger equational description of modal operators. In particular we will show that:

1. If a GAO $(A, \Diamond, \Box)$ satisfies the axioms $(P1)$: $\Box(a \vee b) \leq \Box a \vee \Diamond b$ and $(P2) : \Box a \vee \Diamond b \leq \Diamond(a \wedge b)$, then the unique relation $R = R(\Diamond) \cap R(\Box)$ on $F(A)$ determines two operators $O(R), O'(R)$ on $G(F(A))$ such that $(A, \Diamond, \Box)$ and $(G(F(A)), O(R), O'(R))$ are isomorphic.

2. If a GAO $(A, \Diamond, \Box)$ satisfies the axiom $(D) : \Box x \leq \Diamond x$, then the forest-frame $(F(A), R(\Diamond), R(\Box))$ satisfies $R(\Box) \subseteq R(\Diamond)$ and hence, with respect to what we claimed in (1) the unique relation allowing to represent $(A, \Diamond, \Box)$, is $R = R(\Diamond) \cap R(\Box) = R(\Box)$.

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