Non-Classical Modal and Predicate Logic

23rd – 26th November 2021

organized by the chairs of
Logic and Epistemology and Nonclassical Logic
at the Department of Philosophy I
of Ruhr University Bochum

Book of Abstracts

Location details for physical participation:
Jahrhunderthaus
Alleestraße 80
44973 Bochum

Please direct all inquiries by email to: `ncmpl2021 [at] rub [dot] de`

Tuesday
23rd November

9:15 – 9:30 Introduction

9:30 – 10:30 **Two-layered Belnapian logics for uncertainty**
M. Bílková 9

10:30 – 11:00 **Information Types in Intuitionistic Predicate Logic**
V. Punčochář, C. Noguera 87

11:00 – 11:30 *Coffee/tea break*

11:30 – 12:00 **Paraconsistent modal logic of comparative uncertainty**
M. Bílková, S. Frittella, D. Kozhemiachenko 26

12:00 – 12:30 **Logic with Two-Layered Modal Syntax: Abstract, Abstracter, Abstractest**
P. Cintula, C. Noguera 42

12:30 – 13:00 **Changing the World Constructively**
I. Sedlár 102

13:00 – 14:15 *Lunch break*

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T. Braüner, P. Blackburn, J. L. Kofod 29

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A. Chafik, F. Cheikh-Alili, J.-F. Condotta, I. Varzinczak 36

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J. Raab 90

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D. Fuenmayor 55

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T. Flaminio, L. Godo, P. Menchón, R. O. Rodriguez 52

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J. Raclavský, P. Kuchiňka 93

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A. Dvořák 43

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L. Běhounek 22

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are at Most 8-valued
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in infinitary logics
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G. Greco, P. Jipsen, A. Kurz, M. A. Moshier, A. Palmigiano, A. Tzimoulis 66

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13:30 – 13:45 Closing remarks

13:45 – 15:00 *Lunch*

Invited talks

Contributed talks

On the role of Dunn and Fisher Servi axioms in relational frames for Gödel modal logics

Tommaso Flaminio, Lluis Godo, Paula Menchón and Ricardo Oscar Rodriguez

Extending modal logics to a non-classical propositional ground has been, and still is, a fruitful research line that encompasses several approaches, ideas and methods. In the last years, this topic has significantly impacted on the community of many-valued and mathematical fuzzy logic that have proposed ways to expand fuzzy logics (t-norm based fuzzy logics, in the terminology of Hájek [8]) by modal operators so as to capture modes of truth that can be faithfully described as "graded".

In this line, one of the fuzzy logics that has been an object of major interest without any doubt is the so called *Gödel logic*, i.e., the axiomatic extension of intuitionistic propositional calculus given by the *prelinearity axiom*: $(\varphi \to \psi) \vee (\psi \to \varphi)$. As first observed by Horn in [9], prelinearity implies completeness of Gödel logic with respect to totally ordered Heyting algebras, i.e., *Gödel chains*. Indeed, prelinear Heyting algebras form a proper subvariety of that of Heyting algebras, usually called the variety of Gödel algebras and denoted $\mathbb{G}$ whose subdirectly irreducible elements are totally ordered. Furthermore, in contrast with the intuitionistic case, $\mathbb{G}$ is locally finite, whence the finitely generated free Gödel algebras are finite.

Modal extensions of Gödel logic have been intensively discussed in the literature [2, 3, 10]. Following the usual methodological and philosophical approach to fuzzy logic, they have been mainly approached semantically by generalizing the classical definition of Kripke model $\langle W, R, e\rangle$ by allowing both the evaluation of (modal) formulas and the accessibility relation R to range over a Gödel algebra, rather than the classical two-valued set $\{0, 1\}$ (see [1] for a general approach). More precisely, a model of this kind, besides evaluating formulas in a more general structure than the classical two-element boolean algebra, regards the accessibility relation R as a function from the cartesian product $W \times W$ to a Gödel algebra $\mathbf{A}$ so that, for all $w, w' \in W$, $R(w, w') = a \in A$ means that a is the *degree of accessibility* of w' from w.

Here, we put forward a novel approach to Gödel modal logic that leverages on the duality between finite Gödel algebras and finite forests. This line, that was previously presented in [7], is deepened and extended by the present approach. In particular, we ground our investigation on finite Gödel modal algebras and their dual structures, that is, the prime spectra of finite Gödel algebras ordered by reverse-inclusion. These ordered structures can be regarded as the prelinear version of posets and they are known in the literature as *finite forests*: finite posets whose principal downsets are totally ordered. In general, Gödel algebras

with modal operators form a variety denoted by $\mathbb{GA}\mathbb{O}$ for *Gödel algebras with operators*. Hence, the algebras we are concerned with are those belonging to the finite slice of $\mathbb{GA}\mathbb{O}$. The associated relational structures based on forests, as we briefly recalled above, might hence be regarded as the prelinear version of the usual relational semantics of intuitionistic modal logic. Accessibility relations $R_\Box$ and $R_\Diamond$ on finite forests are defined, in our frames, by ad hoc properties that we express in terms of (anti)monotonicity on the first argument of the relations themselves. These relational frames will be called *forest frames*.

Furthermore, we put forward a comparison between our approach to the ones that have been proposed for intuitionistic modal logic and, in particular, those developed by Palmigiano in [12] and Orłowska and Rewitzky in [11]. By analyzing the role that these different relational frames (namely, those presented by Palmigiano, Orłowska and Rewitzky, and ours) have in proving a Jónsson-Tarski like representation theorem for Gödel algebras with modal operators, we realized that forest frames situate in a middle level of generality between those of Palmigiano and those of Orłowska and Rewitzky. The former being the less and the latter being the more general ones.

More in details, we observe that, if we start from any Gödel algebra with operators $(\mathbf{A}, \Box, \Diamond)$, its associated forest frame $(\mathbf{F}_\mathbf{A}, R_\Box, R_\Diamond)$ allows to construct another algebraic structure $(\mathbf{S}_{\mathbf{F}_\mathbf{A}}, \beta_\Box, \delta_\Diamond)$ isomorphic to the starting one. Interestingly, the forest frame $(\mathbf{F}_\mathbf{A}, R_\Box, R_\Diamond)$ is not the unique one that reconstructs $(\mathbf{A}, \Box, \Diamond)$ up to isomorphisms. Indeed, for every Gödel algebra with operators $(\mathbf{A}, \Box, \Diamond)$, there are non-isomorphic forest frames, Palmigiano-like, and Orłowska and Rewitzky-like frames that determine the same original modal algebra $(\mathbf{A}, \Box, \Diamond)$ up to isomorphism.

We start by considering the most general way to define the operators $\Box$ and $\Diamond$ on Gödel algebras and investigating the relational structures corresponding to the resulting algebraic structures. Later on, we focus on particular and well-known extensions. Precisely we consider two main extensions of Gödel algebras with operators: (1) the first one is obtained by adding the Dunn axioms, typically studied in the fragment of positive classical (and intuitionistic) logic [5, 4]; (2) the second one is determined by adding the Fischer-Servi axioms [6]. From the algebraic perspective, adding these identities to Gödel algebras with operators identifies two proper subvarieties of $\mathbb{GA}\mathbb{O}$ that we respectively denoted by $\mathbb{DGA}\mathbb{O}$ and $\mathbb{FSGA}\mathbb{O}$.

In contrast with the case of general Gödel algebras with operators whose relational structures need two independent relations to treat the modal operators, the structures belonging to $\mathbb{DGA}\mathbb{O}$ and $\mathbb{FSGA}\mathbb{O}$ only need, for their Jónnson-Tarski like representation, frames with only one accessibility relation. In addition, we study in detail the relational structures corresponding to two further subvarieties of $\mathbb{GA}\mathbb{O}$. The first one is the variety obtained as the intersection $\mathbb{DGA}\mathbb{O} \cap \mathbb{FSGA}\mathbb{O}$. The algebras belonging to such variety have been called *bimodal Gödel algebras* in [3] and a modal algebra $(\mathbf{A}, \Box, \Diamond) \in \mathbb{DGA}\mathbb{O} \cap \mathbb{FSGA}\mathbb{O}$ is characterized by the property stating that, for every boolean element $b \in A$, both $\Box b$ and $\Diamond b$ are boolean as well. The second subvariety that we consider refines $\mathbb{DGA}\mathbb{O}$. Indeed, any algebra $(\mathbf{A}, \Box, \Diamond)$ belongs to this class iff it satisfies

Dunn axioms, plus the requirement that $\Box a$ and $\Diamond a$ are boolean for all $a \in A$.

References

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