Preface

A measure assigns values to sets and generalizes the concept of length, area and volume. Probability measures are a well known example of measures. Lebesgue measures are another example. Most typically, measures are additive, as length, areas and volumes are. That is, the measure of the union of two disjoint sets is the sum of these two measures

$$\mu(A \cup B) = \mu(A) + \mu(B)$$

for $A \cap B = \emptyset$. Again, probabilities are an example of additive measures.

Although not so much known, non-additive measures have also been studied in the literature both for their mathematical properties as well as for their application to real problems. Non-additive measures replace additivity my monotonicity. That is,

$$\mu(A) \leq \mu(C) \text{ if } A \subseteq C.$$

As all additive measures are monotonic, non-additive measures generalize additive ones.

Non-additive measures are also known by the term capacities, and fuzzy measures.

Non-additive measures permit us to represent interaction between the elements. For example, we might have $\mu(A \cup B) < \mu(A) + \mu(B)$ (negative interaction between A and B), and $\mu(A \cup B) > \mu(A) + \mu(B)$ (positive interaction between A and B).

Then, in the same way that there are fundamental concepts in measure theory based on additive measures, there are some based on non-additive measures. Some of these concepts are generalizations of corresponding concepts for additive measures. For example, the Choquet integral [1] is a generalization of the Lebesgue integral in the sense that the Choquet integral of a function f with respect to an additive measure corresponds to the Lebesgue integral. Other concepts were introduced as new in this non-additive setting. This is the case of the integral introduced by Sugeno in 1972 [4, 5] which is now known as the Sugeno integral.

This book has its origin in the 9th International Conference on Modeling Decisions for Artificial Intelligence (MDAI 2012) that took place in Girona[1] and, more specifically, in the panel session *Fuzzy measures, fuzzy integrals and aggregation operators* hold in the conference. The panel gathered key researchers with the aim of discussing new and challenging lines for future research in the area of non-additive measures and integrals. The chapters of this book, written by most of the panelists and two additional invited authors, are state-of-the-art descriptions of the field that cover the lines of research discussed in the panel.

The first chapter is a review of uses and applications of non-additive measures and integrals. The chapter presents most relevant definitions and also points out

[1] http://www.mdai.cat/mdai2012

to the other chapters in the book for further details and references. Links between non-additive integrals and aggregation operators [6] are also highlighted.

In the second chapter, Narukawa presents an overview of integration with respect to a non-additive measure. The chapter gives special emphasis to integrals over continuous domains. The Sugeno, Choquet and generalized integrals are presented and their properties reviewed. The case of multidimensional integrals are discussed and a Fubini-like theorem is presented. The chapter concludes with the Möbius transform and generalizations of the Möbius transform.

In the third chapter, Mesiar and Stupňanová focuses on different integrals with respect to a non-additive measure. The authors discuss the approach to integration introduced by Even and Lehrer [2] (decomposition integrals), the Choquet and Sugeno integrals and also Shilkret and universal integrals.

Chapter four, by Honda, focuses on the definition of entropy for non-additive measures (or capacities). First, Honda reviews the definition of entropy for probabilities and then introduces different generalizations that exist for non-additive measures. The author not only considers the case of measures defined on 2^X but also on measures defined on set systems (based on a subset of 2^X). The problem of the axiomatization of entropies is also discussed.

Non-additive measures and integrals have been used in applications. The fifth chapter by Ozaki focuses on their application to economics. More especifically, the author considers decision theory under risk, and decision theory under uncertainty. The chapter describes some of the problems and paradoxes that cannot be solved using additive models (as e.g. Ellsberg's paradox).

Fujimoto in Chapter six surveys cooperative game theory, an important application area for non-additive measures. Non-additive measures permit to represent coalitions in game theory. The chapter discusses with detail the case in which not all coalitions can be formed, and how we can deal with this situation. The chapter also discusses indices that have been defined for games (as the Shapley index [3]).

Flaminio, Godo, and Kroupa focus in Chapter seven on belief functions on MV-algebras of fuzzy sets. Belief functions are totally monotone non-additive measures. The authors discuss two ways of extending belief functions on Boolean algebras of events to MV-algebras of events.

We hope that this book will provide a reference to students, researchers and practitioners in the field.

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