2nd. Part

- Modeling
  - Primality/Duality
  - Global Constraints
- Constraint programming
  - examples in CHOCO
- Soft Constraints
  - Models
  - Algorithms

Modeling

- Any CSP can be formulated in different (equivalent) ways
- The efficiency of the solving algorithms can vary dramatically
- No strong results are known
- Active line of research
- Alternative formulations:
  - Primal/Dual
  - Primitive/Global constraints

Primal/Dual

Primal CSP: (X, D, C)

- $X = \{x_1, x_2, \ldots, x_n\}$, $D = \{d_1, d_2, \ldots, d_n\}$, $C = \{c_1, c_2, \ldots, c_r\}$

$c \in C$ $\quad$ $var(c) = \{x_i, x_j, \ldots, x_k\}$ $\quad$ *scope*

$rel(c) \subseteq d_i \times d_j \times .. \times d_k$ $\quad$ *permitted tuples*

Primal/Dual

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$rel(c) \subseteq d_i \times d_j \times .. \times d_k$ $\quad$ *permitted tuples*

Dual CSP: (X', D', C')

- $X' = \{x'_1, x'_2, \ldots, x'_r\}$,
- $D' = \{d'_1, d'_2, \ldots, d'_r\}$, where $d'_i = rel(c_i)$
- $C' = \{c'_{ij}\}$, binary constraints

  $var(c'_{ij}) = \{x_i, x_j\}$

  $\exists\, c'_{ij} \in C' \iff rel(c_i) \cap rel(c_j) \neq \emptyset$

  $rel(c'_{ij})$ = consistent pairs of tuples

Example: Crossword puzzles

1	2	3	4	5
6	7	8		9
10	11	12	13	14
15		16	17	18
19	20	21	22	23

a
aardvark
aback
abacus
abaft
abalone
abandon
...

Mona Lisa
monarch
monarchy
monarda
...
zymurgy
zyrian
zythum

Primal model (Non-binary)

1	2	3	4	5
6	7	8		9
10	11	12	13	14
15		16	17	18
19	20	21	22	23

- variables:
  one for each unknown letter (cell)
- domains:
  ‘a’, …, ‘z’
- constraints:
  contiguous letters must form words in dictionary

Dual model (binary)

1	2	3	4	5
6	7	8		9
10	11	12	13	14
15		16	17	18
19	20	21	22	23

- variables:
  one for each unknown
  word across and down
- domains:
  words from dictionary
- constraints
  intersecting words must
  agree on common letter

Global Constraints

c is global iff:

- arity(c)=$r > 2$
- c is logically equivalent to $\{c_1,c_2,\ldots,c_k\}$ binary
- **AC**(c) *prunes more* than **AC**($c_1,c_2,\ldots,c_k$)

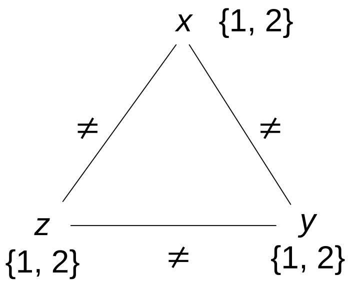
Propagation:

- There is a specialized efficient algorithm (exploits the semantics)

Catalog:

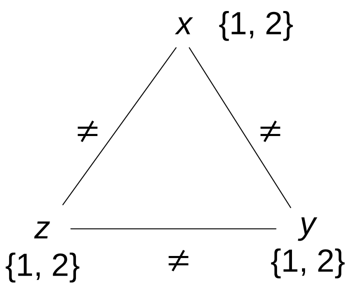
- set of global constraints
- best known algorithms for propagation

Example: all-different

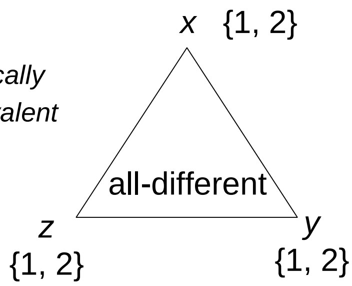


3 binary constraints,
they are AC,
no pruning

Example: all-different



logically equivalent



3 binary constraints,
they are AC,
no pruning

1 ternary constraint,
it is not AC,
AC pruning → empty domain
no solution!!

Example: all-different

- Enforcing arc-consistency:
  - n variables, d values
  - $n(n-1)/2$ binary constraints : $O(n^2 d^2)$
  - 1 n-ary constraint:
    - general purpose algorithm $O(d^n)$
    - specialized algorithm $O(n^2 d^2)$

Constraint Programmming

Declarative Programming: you declare

- Variables
- Domains
- Constraints

and ask the SOLVER to find a solution!!

SOLVER offers:

- Implementation for variables / domains / constraints
- Hybrid algorithm: backtracking + incomplete inference
- Global constraints + optimized AC propagation
- Empty domain detection
- Embedded heuristics

Constraint Logic Programming

- Logic Programming:
  - implements chronological backtracking
- Constraint logic programming:
  - extension including constraint satisfaction facilities
- Existing solvers:
  - Chip (`www.cosytec.com`)
  - Eclipse (`www-icparc.doc.ic.ac.uk/eclipse`)
  - Sicstus Prolog (`www.sics.se/sicstus`)
  - ...

Imperative Constraint Programming

Library to be included in your (procedural) program

Provides:

- Special objects:
  - Variables / Domains / Constraints (global)
- Special functions to find:
  - One solution / the next solution

- Existing Solvers:
  - Ilog Solver (`www.ilog.com`)
  - Choco (`www.choco-constraints.net`)

CHOCO

- Library for modeling and solving combinatorial problems
- Intended for academic purposes
- Plus:
  - Free software (GPL from FSF)
  - Simple
  - Efficient
  - Generic
- Minus:
  - Implemented in Claire (which is implemented in C++)
  - Not (completely) stable

Choco: 1st example

```
[sillyCSP() : void
 -> let pb := choco/makeProblem("Silly CSP",3),
        x := choco/makeIntVar(pb, "x", 1, 3),
        y := choco/makeIntVar(pb, "y", 1, 3),
        z := choco/makeIntVar(pb, "z", 1, 3) in
        (choco/post(pb, x + y == z),
        choco/post(pb, x > y),
        choco/solve(pb,false),
        printf("~S ~S ~S\n",x,y,z) )]
```

Choco: 2nd example

```
[queens(n:integer, all:boolean)
 -> let pb := choco/makeProblem(" n queens",n),
     queens := list{choco/makeIntVar(pb,"Q" /+ string!(i), 1, n) | i in (1 .. n) }
    in
     (for i in (1 .. n)
        for j in (i + 1 .. n)
           let k := j - i in
              ( choco/post(pb, queens[i] !== queens[j]),
                choco/post(pb, queens[i] !== queens[j] + k),
                choco/post(pb, queens[j] !== queens[i] + k) ),
      choco/solve(pb,all) )]
```

Soft Constraints

- Motivation
- Models:
  - Fuzzy CSP
  - Weighted CSP
  - Valued CSP
- Algorithms:
  - Search
  - Dynamic programming
  - Approximate algorithms

Motivation

- Using the classical CSP framework:
  - Many problems have many solutions
    - Algorithms either give the first one they find or all of them
    - Typically, the user likes some solutions more than others
  - Many problems do not have any solution
    - Algorithms just report failure
    - Typically, the user can identify some non critical constraint

Soft CSP

- Problems:
  - Variables and domains as in classical CSP
  - *Mandatory* constraints (*hard*)
  - *Preference* constraints (*soft*)
- Feasible solution:
  - Complete assignment which satisfies every hard constraint
- Optimal solution:
  - Preferred feasible solution, according to soft constraints
- Complexity:
  - Np-hard
  - Much harder than classical CSP

Soft Constraints Models

- Max-csp [freuder and wallace 92]
- Fuzzy CSP [dubois et al 93]
- Lexicographic CSP [fargier et al 93]
- Weighted CSP
- Probabilistic CSP [fargier and lang 93]
- Valued CSP [schiex et al 95]
- Semiring-based CSP [bistarelli et al 95]

Notación

- Variables: i, j, k, ...
- Domains: D_i, D_j, ...
- Values: a,b,...
- (Binary) constraints: c_{ij}
- Tuples: τ
- Projection: $\tau_{[i,j]}$

Classical CSP

- Expressable as classical logic
- Constraints: boolean functions
  - $C_{ij}(a,b)$= *true/false*

- Task of interest:

$$\exists \tau \, \forall c_{ij} \quad c_{ij}(\tau_{[i,j]})$$

Fuzzy CSP

- Extension of classical CSP to *fuzzy logic*
  - Conjunction: t-norm (*mínimum*)
  - Disjunction: t-conorm (*maximum*)
  - $c_{ij}(a,b) \in [0,1]$

  - Task:

$$\max_{\tau}\{\min_{c_{ij}}\{c_{ij}(\tau_{[i,j]})\}\}$$

Weighted CSP

- Preferences are expressed as *costs*
  - Constraints: cost functions

$$c_{ij}(a,b) \in \{0,1,\ldots,\infty\}$$

  - Task:

$$\min_{\tau}\{\sum_{c_{ij}} \{c_{ij}(\tau_{[i,j]})\}\}$$

Example

- Airlines flight scheduling:
  - Input:
    - Aircrafts, airports
    - Flights: (origin, destination, frequency)
    - Requirements:
      - No more than four legs per flight
      - 1 hour < transfer time < 5 hours
      - ...
  - Output:
    - Schedule: each flight is a sequence of scheduled legs

Example

- Classical CSP:
  - Consistent schedules
- Fuzzy CSP:
  - Schedules where every flight is reasonably good
    - Maximizes the quality of the worst flight
- Weighted CSP:
  - Schedules where, globally, flights are good
    - Maximizes the sum of qualities over flights
    - Some flights can be very inconvenient

Valued CSP (VCSP)

[Schiex *et al* 95]

- Axiomatic model aiming at maximal generality
- It includes all previous models
- Valuation structure (E,$*$,>):
  - E is the set of *valuations*
    - Totally ordered by ">", the maximum element is "$\top$", the minimum element is "$\perp$".
  - $*$ is the *aggregation* of valuations
    - *binary* operation on E, *commutative* and *associative*.
    - $\perp$ is the *identity*
    - $\top$ is *absorbing*
    - $*$ grows *monotonicly*

Valued CSP

- (Soft) constraints:
  - $c_{ij}(a,b) \in E$
- Task:
  - $\min_{\tau}\{ *_{c_{ij}} \{c_{ij}(\tau_{[i,j]})\}\}$

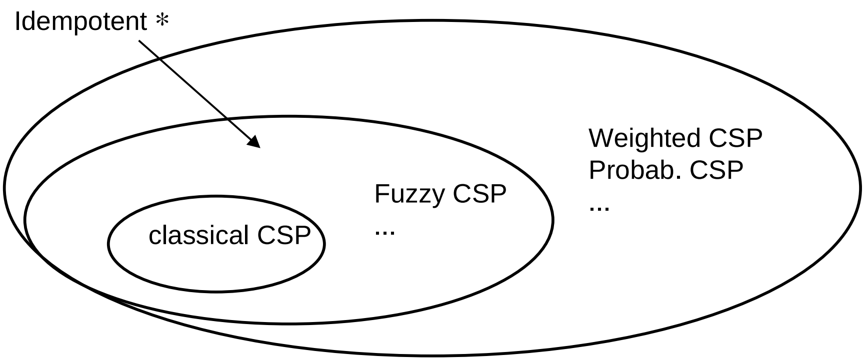
Soft CSP Solving

- Preliminaries
- Exact algorithms:
  - Branch and Bound
  - Partial Forward Checking
  - Reversible Dacs
  - Russian Doll Search
  - Bucket Elimination

tree search

- Approximate algorithms:
  - Local search approaches
  - Interval approximation

Valued CSP



Solving Valued CSP

- Classical CSP:
  - First part of this tutorial
  - Forward checking, *k*-consistency, MAC, ...
- Idempotent CSP:
  - Algorithms and properties from classical CSP extend easily
- Non idempotent CSP:
  - Algorithms and properties from classical CSP do not extend so easily
  - Last part of this tutorial
  - For simplicity, we will consider Max-csp