CSP: An Introduction

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Overview

Definitions

Examples

- Map colouring
- N-queens
- Car sequencing
- Job-shop scheduling

Relevance

Constraint graphs

Some Definitions

Constraint Network (CN): (X, D, C)

- $X = \{x_1, x_2, \ldots, x_n\}$ variables
- $D = \{d_1, d_2, \ldots, d_n\}$ domains (finite)
- $C = \{c_1, c_2, \ldots, c_r\}$ constraints

$c \in C$ $\quad var(c) = \{x_i, x_j, \ldots, x_k\}$ *scope*

$rel(c) \subseteq d_i \times d_j \times .. \times d_k$ *permitted tuples*

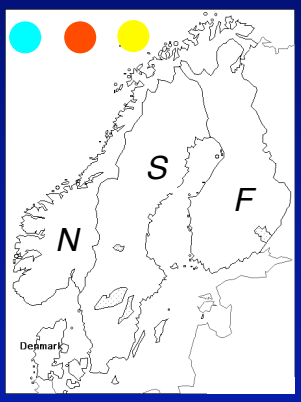
$arity(c) = |var(c)|$ (*unary, binary, ternary,…*)

Constraint Satisfaction Problem (CSP):

- CN solving: assig. satisfying every constraint
- NP-complete task

Example: Map Colouring

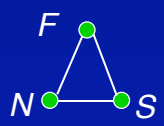
GOAL: Given a map and a number of colours, assign a colour to each region such that adjacent regions have different colours



Formulation:

- Variables: regions
- Domains: colours
- Constraints: if *adjacent*(x_i, x_j) then $x_i \neq x_j$

Constraint Graph:



Example: n-queens

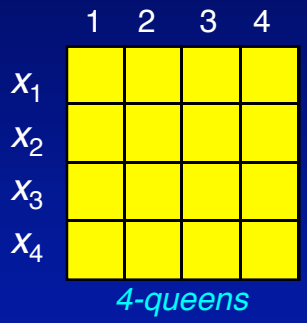
GOAL: Locate *n* queens in an *n* x *n* chessboard, such that they do not attack each other

Formulation:

- Variables: one queen per row
- Domains: available columns
- Constraints:

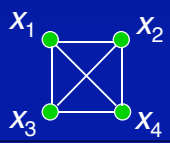
  different columns and different diagonals

  $x_i \neq x_j$ $\quad |x_i - x_j| \neq |i - j|$



4-queens

Constraint Graph:



Example: Car Sequencing

OPTIONS / MODELS				
anti-fog	x		x	
sun-roof	x	x		x
climatiser		x	x	

GOAL: ordering satisfying capacity constraints

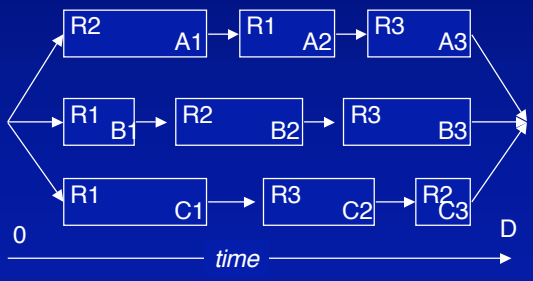
Formulation:

- variables: *n* cars to produce
- domains: car models
- constraints: box capacity

Job-Shop Scheduling

GIVEN: *n* jobs, each with *m* operations
- *m* resources, each operation requires a resouce for a period
- precedence between operations of each job

GOAL: can *n* jobs be performed in time D?



Formulation:
- variables: operations
- dominios: start times
- constraints:
  - precedence
  - exclusivity

Relevance

CSP: formal model to express problems

- Artificial Intelligence
  - temporal reasoning
- Control Theory
  - controllers for sensory based robots
- Concurency
  - process comm. and synchr.
- Computer Graphics
  - geometric coherence
- Database Systems
  - constraint databases
- Bioinformatics
  - sequence alignment
- Operations research
  - optimization

Real-life applications
- Production planning
- Staff scheduling
- Resource allocation
- Circuit design
- Option trading
- DNA sequencing
- ...

Constraint Graphs

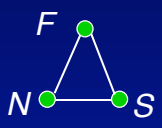
Primal graph:
- Nodes: variables
- Arcs: between two constrained variables

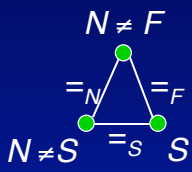
Dual graph:
- Nodes: constraints
- Arcs: between two constraints sharing a variable

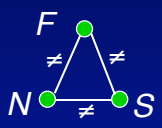
Hypergraph:
- Nodes: variables
- Hyperarcs: constraints

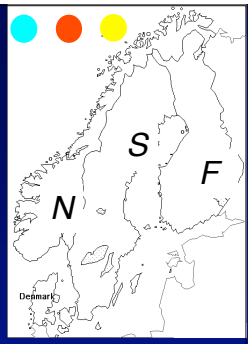
Example: Map Colouring

$N \neq S \neq F \neq N$









all-different (*N*,*S*,*F*)