A Propagation and Aggregation Algorithm for Inferring Opinions in Structural Graphs

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May 10, 2010
Problem Definition: there is a variety of opinions other than explicit ones, and some influential ones may be about related entities.
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**Problem Definition:** there is a *variety* of opinions other than explicit ones, and some influential ones may be about *related* entities.

**Proposed Approach:** dealing with new *categories* of opinions:

- ‘Objective’ opinions
- ‘Subjective’ opinions
  - Direct opinions
  - *Propagated* opinions

\[
\alpha_\beta (\alpha) = \begin{cases} 
\text{bad} = 0.1, \\
\text{good} = 0.6, \\
v.\text{good} = 0.3 
\end{cases}
\]
### Use Cases

<table>
<thead>
<tr>
<th>Uses Structural Graphs</th>
<th>Doesn’t Use Structural Graphs</th>
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<td>Uses Objective Opinions</td>
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Use Cases: Liquid Publications

- Scientific contributions make heavy use of structural graphs.
- Subjective opinions provided by researchers' reviews.
Use Cases: Chess

- There is no clear use of structural graphs.
- Objective opinions are provided by the match scores.
Preliminaries: Structural Graphs

Definition 1. A Structural Graph

A structural graph is defined as the tuple specified by a set of nodes and relations, accordingly:

\[ \text{SG} = \langle N, G, O, E, A, T, \mathcal{P}, \mathcal{F} \rangle \]

where

- \( N \) is the set of nodes/entities,
- \( G \) is the set of agents, peers, people, or even entities that may form opinions about \( \alpha \in N \)
- \( O \) is the set of direct opinions
- \( E \) is the evaluation space
- \( A \) is the set of attributes
- \( T \) represents calendar time,
- \( \mathcal{P} \subseteq N \times N \) specifies which nodes are part of the structure of which others,
- \( \mathcal{F} : R \times N \times A \times T \rightarrow O \) links a given agent, node, attribute, and time to an opinion.
Preliminaries: Individual Opinion

**Definition 2. An Opinion**

The opinion that an entity $\beta$ may hold about entity $\alpha$ at time $t$ is specified as a probability distribution over the evaluation space $E$, accordingly:

$$o^t_\beta(\alpha) = \{e_1 \mapsto v_1, \ldots, e_n \mapsto v_n\}$$

where

- $E = \{e_1, \ldots, e_n\}$, and
- $v_i \in [0, 1]$, such that $\sum_i v_i = 1$. 
Preliminaries: Group Opinion

Definition 3. **Objective Group Opinion** (D)
Objective group opinion is based on objective opinions.

Definition 4. **Subjective Group Opinion** (P)
Subjective group opinion is based on objective and subjective opinions.

- Objective opinions are more reliable than subjective ones.
- Objective group opinion is more reliable than subjective one.
- Subjective group opinion decays towards the objective one.
Reputation, Reliability, and Ranking

Reputation, or reliability\(^\dagger\), depends on how far the objective group opinion is from the ideal target \(T\):

\[
R^t_\alpha = 1 - \text{EMD}(D^t_\alpha, T) \tag{1}
\]

\(R\) may then be used to rank entities, accordingly.

\(^\dagger\)Equating reputation to reliability is based on \textit{ex-cathedra} argument.
Objective Group Opinion: \( \mathbb{D} \)

- \( \mathbb{D}_\alpha^t \) is an aggregation of all the objective opinions formed about \( \alpha \) by the time \( t \), that takes into consideration the impact of each opinion:

\[
\mathbb{D}_\alpha^t = \frac{\sum_{\beta \in G} \sum_{t' \in T_\beta(\alpha)} \theta_{\beta}^{t'\rightarrow t}(\alpha) \cdot \mathcal{I}(\theta_{\beta}^{t'\rightarrow t}(\alpha))}{\sum_{\beta \in G} \sum_{t' \in T_\beta(\alpha)} \mathcal{I}(\theta_{\beta}^{t'\rightarrow t}(\alpha))} \tag{2}
\]

- The impact of an opinion depends on how informative it is:
  i.e. the difference in entropy between the opinion and the flat distribution

\[
\mathcal{I}(\theta_{\beta}^t(\alpha)) = \mathcal{H}(\theta_{\beta}^t(\alpha)) - \mathcal{H}(\mathbb{F}) \tag{3}
\]

- The value of an opinion depends on the reliability of its holder:

\[
\theta_{\beta}^t(\alpha) = \mathcal{R}_{\beta}^t \times \phi_{\beta}^t(\alpha) + (1 - \mathcal{R}_{\beta}^t) \times \mathbb{F} \tag{4}
\]
Objective Group Opinion: \( D \)

- Initially, there is ignorance: \( D_0^\alpha = F \)

- With time, everything decays towards ignorance:
  \[
  \lim_{t \to \infty} D_t^\alpha = F \quad \text{and} \quad \lim_{t \to \infty} O_t^\beta(\alpha) = F
  \]
Subjective Group Opinion: \( \mathbb{P} \)

- A new **subjective** opinion \( \mathcal{X} \) is aggregated to the existing subjective group opinion \( \mathbb{P} \):

\[
\mathbb{P}^t_{\alpha} = \zeta \mathbb{P}^{t'}_{\alpha} + (1 - \zeta) \mathcal{X}
\]

(5)

\( \mathcal{X} \) may represent:
- direct subjective opinion \( \mathcal{O}^{t}_{\beta}(\alpha) \)
- group opinion \( \mathbb{P} \) about a neighbouring node \( \beta \) (the propagation case)

\( \zeta \) generally represents the reliability of \( \alpha \), but also consider the reliability of the source \( \beta \): \( \zeta = (R^t_{\alpha})R^t_{\beta} \)

- In the case of opinion propagation, the higher the degree of the node \( d_\alpha \), the less influenced it is by one of its neighbours: \( R^t_{\beta} \mapsto (R^t_{\beta} + d_\alpha - 1)/d_\alpha \)
Subjective Group Opinion: \( P \)

- A new **subjective** opinion \( \mathcal{X} \) is aggregated to the existing subjective group opinion \( P \):

\[
P^t_\alpha = \zeta P^{t' \rightarrow t}_\alpha + (1 - \zeta) \mathcal{X}
\]  

(5)

\( \mathcal{X} \) may represent:

- direct subjective opinion \( o^t_\beta(\alpha) \)
- group opinion \( P \) about a neighbouring node \( \beta \) (the **propagation** case)

\( \zeta \) generally represents the reliability of \( \alpha \), but also consider the reliability of the source \( \beta \): \( \zeta = (R^t_\alpha R^t_\beta) \)

- In the case of opinion propagation, the higher the degree of the node \( d_\alpha \), the less influenced it is by one of its neighbours: \( R^t_\beta \mapsto (R^t_\beta + d_\alpha - 1)/d_\alpha \)

- A new **objective** opinion is also aggregated to the existing subjective group opinion \( P \), but its **influence** is stronger than subjective opinions:

\[
P^t_\alpha = \frac{R^t_\alpha P^{t' \rightarrow t}_\alpha + R^t_\beta o^t_\beta(\alpha)}{R^t_\alpha + R^t_\beta}
\]  

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Subjective Group Opinion:

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(6)
Subjective Group Opinion: \( P \)

- Initially, there is ignorance: \( P^0_\alpha = F \)
- With time, inferred opinions tend towards default ones:
  \[
  \lim_{t \to \infty} P^t_\alpha = D^t_\alpha
  \]
Predicting Results of a Chess Game

- **Real Data:**
  - players’ performance is almost random
  - proposed algorithm performs 2.3% better than ELO

- **Simulated Data:**
  - improved performance is noted when there is a change in behavior along time

![Graph showing performance of ELO and Our algorithm over years](image)
Future Work

Application to different categories of use cases:

- Liquid Publications
  - makes use of structural graphs
  - makes use of subjective opinions

- Diplomacy Games
  - does not make use of structural graphs
  - makes use of objective opinions

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  - Publications
  - Chess
- Preliminaries
  - Structural Graphs
  - Individual Opinion
  - Group Opinion
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  - The 3 Rs
  - Objective
  - Subjective
- Results
- Future Work
Thank you