Hierarchies of probability logics

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ECAI 2012 Workshop on Weighted Logics for AI
Montpellier, August 28, 2012.
Outline

- Probabilistic logics, overview
- Hierarchies of PLs
- Conclusion
History (1)

- Leibnitz (1646 – 1716)
- Bernoullies, Bayes, Lambert, Bolzano, De Morgan, MacColl, Peirce, Poretskiy, . . .
- Laplace (1749 – 1827)
- George Boole (1815 – 1864), An Investigation into the Laws of Thought, on which are founded the Mathematical Theories of Logic and Probabilities (1854):

Logical functions:

\[ f_1(x_1, \ldots, x_m), \ldots, f_k(x_1, \ldots, x_m), \]

Probabilities:

\[ p_1 = \Pr(f_1(x_1, \ldots, x_m)), \ldots, p_k = \Pr(f_k(x_1, \ldots, x_m)) \]

Solve:

\[ \Pr(F(x_1, \ldots, x_m)) \text{ using } p_1, \ldots, p_k \]

Corrected by T. Hailperin ('80.)
Leibnitz (1646 – 1716)

Bernoullies, Bayes, Lambert, Bolzano, De Morgan, MacColl, Peirce, Poretskiy, . . .

Laplace (1749 – 1827)

George Boole (1815 – 1864), An Investigation into the Laws of Thought, on which are founded the Mathematical Theories of Logic and Probabilities (1854):

- logical functions: $f_1(x_1, \ldots, x_m), \ldots, f_k(x_1, \ldots, x_m), F(x_1, \ldots, x_m)$
- probabilities:
  - $p_1 = P(f_1(x_1, \ldots, x_m)), \ldots, p_k = P(f_k(x_1, \ldots, x_m))$
- solve: $P(F(x_1, \ldots, x_m))$ using $p_1, \ldots, p_k$

corrected by T. Hailperin (’80.)
• progress of theories concerning derivations of truth in Math. logic
• measure theory, formal calculus of probability, Kolmogorov
• Keynes, Reichenbach, De Finetti, Carnap, Cox, . . .
History (3)

- progress of theories concerning derivations of truth in Math. logic
- measure theory, formal calculus of probability, Kolmogorov
- Keynes, Reichenbach, De Finetti, Carnap, Cox, ... 

- '60, '70: Keisler, Geifmann, Scott
- '80: applications in AI
Degrees of beliefs

- The probability that a particular bird $A$ flies is at least 0.75.

- $P_{\geq 0.75} \text{Fly}(A)$
Early papers

Motivating example (1)

Example

Knowledge base:

- if $A_1$ then $B_1$
- if $A_2$ then $B_2$
- if $A_3$ then $B_3$
- ...
Motivating example (1)

Example

Knowledge base:

if $A_1$ then $B_1$ (cf $c_1$)
if $A_2$ then $B_2$ (cf $c_2$)
if $A_3$ then $B_3$ (cf $c_3$)

... 

Uncertain knowledge: from statistics, our experiences and beliefs, etc.
Motivating example (1)

Example

Knowledge base:

- if \( A_1 \) then \( B_1 \) (cf \( c_1 \))
- if \( A_2 \) then \( B_2 \) (cf \( c_2 \))
- if \( A_3 \) then \( B_3 \) (cf \( c_3 \))

... 

Uncertain knowledge: from statistics, our experiences and beliefs, etc.

- To check consistency of (finite) sets of sentences.
- To deduce probabilities of conclusions from uncertain premisses.
The probabilistic logics allow strict reasoning about probabilities using well-defined syntax and semantics.

Formulas in these logics remain either true or false.

Formulas do not have probabilistic (numerical) truth values.
Formal language

- $\text{Var} = \{p, q, r, \ldots\}$, connectives $\lnot$ and $\land$ $+$
  
  $P \geq s$, $s \in Q \cap [0, 1]$

- $\text{For}_C$ - the set of classical propositional formulas
Formal language

- \( \text{Var} = \{ p, q, r, \ldots \} \), connectives \( \neg \) and \( \wedge \) \( + \)

\[ P \geq s, \quad s \in Q \cap [0, 1] \]

- \( \text{For}_C \) - the set of classical propositional formulas

- Basic probabilistic formula:
  \[ P \geq s \alpha \]

  for \( \alpha \in \text{For}_C \), \( s \in Q \cap [0, 1] \)

- \( \text{For}_P \) - Boolean combinations of basic probabilistic formulas

- \( P < s \alpha \) means \( \neg P \geq s \alpha \), \( \ldots \)
Formal language

- \( \text{Var} = \{ p, q, r, \ldots \} \), connectives \( \neg \) and \( \land \) \( \lor \)

\[ P \geq s, \quad s \in Q \cap [0, 1] \]

- \( \text{For}_C \) - the set of classical propositional formulas

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  for \( \alpha \in \text{For}_C, \ s \in Q \cap [0, 1] \)

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- \( P < s \alpha \) means \( \neg P \geq s \alpha \), \ldots

- \( (P \geq s \alpha \land P < t (\alpha \rightarrow \beta)) \rightarrow P = r \beta \)
Formal language

- \( \text{Var} = \{p, q, r, \ldots\} \), connectives \( \neg \) and \( \wedge \) and \( \vee \)

\[
P_{\geq s}, \quad s \in Q \cap [0, 1]
\]

- \( \text{For}_C \) - the set of classical propositional formulas

- Basic probabilistic formula:

\[
P_{\geq s} \alpha
\]

for \( \alpha \in \text{For}_C \), \( s \in Q \cap [0, 1] \)

- \( \text{For}_P \) - Boolean combinations of basic probabilistic formulas

- \( P_{< s} \alpha \) means \( \neg P_{\geq s} \alpha \), \( \ldots \)

- \( (P_{\geq s} \alpha \land P_{< t}(\alpha \rightarrow \beta)) \rightarrow P_{= r} \beta \)

- \( P_{\geq s} P_{\geq t} \alpha, \beta \lor P_{\geq s} \alpha \notin \text{For} \)
Semantics (1)

A probabilistic model $M = \langle W, H, \mu, v \rangle$:
- $W$ is a nonempty set of elements called worlds,
- $H$ is an algebra of subsets of $W$,
- $\mu : H \to [0, 1]$ is a finitely additive probability measure, and
- $v : W \times \text{Var} \to \{\top, \bot\}$ is a valuation.
Semantics (1)

A probabilistic model $M = \langle W, H, \mu, v \rangle$:

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Measurable models

- $\alpha \in \text{For}_C$
- $[\alpha] = \{w \in W : w \models \alpha\}$
- $[\alpha] \in H$
Hierarchies of probability logics

Semantics (2)

\[ M([P\land Q]) = \frac{1}{3} \]

\[ M([P]) = \frac{4}{5} \]

\[ M([Q]) = \frac{1}{2} \]
Satisfiability (1)

- if $\alpha \in \text{For}_C$, $M \models \alpha$ if $(\forall w \in W) \nu(w)(\alpha) = \top$
- $M \models P_{\geq s}\alpha$ if $\mu([\alpha]_M) \geq s$,
- if $A \in \text{For}_P$, $M \models \neg A$ if $M \not\models A$,
- if $A, B \in \text{For}_P$, $M \models A \land B$ if $M \models A$ and $M \models B$.

A set of formulas $F = \{A_1, A_2, \ldots\}$ is satisfiable if there is a model $M$, $M \models A_i, \ i = 1, 2, \ldots$. 

Satisfiability (2)

\[ M \models p \]
\[ M \not\models \neg p \]
\[ M \not\models q \]
\[ M \models p \lor q \]
\[ M \models p \land \left( p \geq 0.5 \right) \land \left( p \leq 0.9 \right) \]

\[ M \models \neg q \]

\[ M \models \neg p \]

\[ M \not\models \neg q \]

\[ M \models \left( p \geq 0.5 \right) \land \left( p \leq 0.9 \right) \]

\[ \mu_{w_1} = \frac{5}{6} \]

\[ \mu_{w_2} = \frac{1}{3} \]

\[ \mu_{w_3} = \frac{1}{2} \]

\[ \mu_{w_4} = \frac{1}{6} \]

\[ \mu_{w_5} = \frac{1}{6} \]

\[ \mu_{w_6} = \frac{1}{6} \]
Logical issues (1)

- Providing a sound and complete axiom system
  - simple completeness (every consistent formula is satisfiable, $\models A$ iff $\vdash A$)
  - extended completeness (every consistent set of formulas is satisfiable)
- Decidability (there is a procedure which decides if an arbitrary formula formula is valid)
Logical issues (1)

- Providing a sound and complete axiom system
  - simple completeness (every consistent formula is satisfiable, \( \models A \iff \vdash A \))
  - extended completeness (every consistent set of formulas is satisfiable)
- Decidability (there is a procedure which decides if an arbitrary formula formula is valid)
- Compactness (a set of formulas is satisfiable iff every finite subset is satisfiable).
Logical issues (2)

- Inherent non-compactness:

\[ F = \{ \neg P = 0 \} \cup \{ P < 1/n : n \text{ is a positive integer} \} \]

\[ F_k = \{ \neg P = 0 \} \cup \{ P < 1/n : n \text{ is a positive integer} \} \]

\[ c : 0 < c < 1/k, \quad \mu_p = c \]

\[ M \text{satisfies every } F_k, \text{ but does not satisfy } \frac{\text{finitary axiomatization + extended completeness}}{\Rightarrow} \text{compactness} \]

Finitary axiomatization for real valued probabilistic logics: there are consistent sets that are not satisfiable
Logical issues (2)

- Inherent non-compactness:

  \[ F = \{ \neg P = 0 \} \cup \{ P < 1/n p : n \text{ is a positive integer} \} \]

- \( F_k = \{ \neg P = 0, P < 1/1p, P < 1/2p, \ldots, P < 1/kp \} \)

- \( c: 0 < c < \frac{1}{k}, \quad \mu[p] = c \)

- \( M \) satisfies every \( F_k \), but does not satisfy \( F \)
Logically issues (2)

- Inherent non-compactness:
  
  \[ F = \{ \neg P = 0 p \} \cup \{ P < 1/n p : n \text{ is a positive integer} \} \]

- \( F_k = \{ \neg P = 0 p, P < 1/1 p, P < 1/2 p, \ldots, P < 1/k p \} \)

- \( c: 0 < c < \frac{1}{k}, \quad \mu[p] = c \)

- \( M \) satisfies every \( F_k \), but does not satisfy \( F \)

- finitary axiomatization + extended completeness \( \Rightarrow \) compactness

- finitary axiomatization for real valued probabilistic logics: there are consistent sets that are not satisfiable
Logical issues (3)

- Restrictions on ranges of probabilities: \( \{0, \frac{1}{n}, \frac{2}{n}, \ldots, \frac{n-1}{n}, 1\} \)

- Infinitary axiomatization
\( LPP_{2}^{Fr(n)} \)

\[ LPP_{2}^{Fr(n)}, \mu : H \rightarrow \{0, \frac{1}{n}, \frac{2}{n}, \ldots, \frac{n-1}{n}, 1\} \]
\[ \text{LPP}^\text{Fr}(n) \]

\[ LPP^\text{Fr}(n), \mu : H \rightarrow \{0, \frac{1}{n}, \frac{2}{n}, \ldots, \frac{n-1}{n}, 1\} \]

\[ \models_{LPP^\text{Fr}(n)} P > \frac{k}{n} p \rightarrow P \geq \frac{k+1}{n} p \]
\( LPP_2^{Fr(n)} \)

- \( LPP_2^{Fr(n)} \), \( \mu : H \rightarrow \{0, \frac{1}{n}, \frac{2}{n}, \ldots, \frac{n-1}{n}, 1\} \)

\[
\models_{LPP_2^{Fr(n)}} P > \frac{k}{n} p \rightarrow P \geq \frac{k+1}{n} p
\]

- \( n = 2, LPP_2^{Fr(2)} \), \( \mu : H \rightarrow \{0, \frac{1}{2}, 1\} \)

\[
\models_{LPP_2^{Fr(2)}} P > \frac{1}{2} p \rightarrow P \geq \frac{1+1}{2} p
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\[ LPP_{2}^{Fr(n)} \]

- \( LPP_{2}^{Fr(n)} \), \( \mu : H \rightarrow \{0, \frac{1}{n}, \frac{2}{n}, \ldots, \frac{n-1}{n}, 1\} \)

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- \( n = 2, \ LPP_{2}^{Fr(2)} \), \( \mu : H \rightarrow \{0, \frac{1}{2}, 1\} \)

\[ \models_{LPP_{2}^{Fr(2)}} P > \frac{1}{2} p \rightarrow P \geq \frac{1+1}{2} p \]

- \( m = 3, \ LPP_{2}^{Fr(3)} \), \( \mu : H \rightarrow \{0, \frac{1}{3}, \frac{2}{3}, 1\} \), \( \mu(p) = \frac{2}{3} \)

\[ \not\models_{LPP_{2}^{Fr(3)}} P > \frac{1}{2} p \rightarrow P \geq \frac{1+1}{2} p \]
Axioms

- all instances of classical propositional tautologies
- axioms for probabilistic reasoning
  - $P \geq 0 \alpha$
  - $P \leq r \alpha \rightarrow P < s \alpha, s > r$
  - $P < s \alpha \rightarrow P \leq s \alpha$
  - $(P \geq r \alpha \land P \geq s \beta \land P \geq 1 (\neg (\alpha \land \beta))) \rightarrow P \geq \min(1, r + s) (\alpha \lor \beta)$
  - $(P \leq r \alpha \land P < s \beta) \rightarrow P < r + s (\alpha \lor \beta), r + s \leq 1$
**LPP_2 (2)**

Rules

- From $\Phi$ and $\Phi \rightarrow \Psi$ infer $\Psi$.
- From $\alpha$ infer $P_{\geq 1}\alpha$.
- From $\{A \rightarrow P_{\geq s - \frac{1}{k}}\alpha, \text{ for } k \geq \frac{1}{s}\}$ infer $A \rightarrow P_{\geq s}\alpha$. 
Proof from the set of formulas ($F \vdash \varphi$):
- at most denumerable sequence of formulas $\varphi_0, \varphi_1, \ldots, \varphi$,
- $\varphi_i$ is an axiom or a formula from the set $F$,
- or $\varphi_i$ is derived from the preceding formulas by an inference rule.

A formula $\varphi$ is a theorem ($\vdash \varphi$) if it is deducible from the empty set.
Proof from the set of formulas \((F \vdash \varphi)\):

- at most denumerable sequence of formulas \(\varphi_0, \varphi_1, \ldots, \varphi\),
- \(\varphi_i\) is an axiom or a formula from the set \(F\),
- or \(\varphi_i\) is derived from the preceding formulas by an inference rule

A formula \(\varphi\) is a theorem \((\vdash \varphi)\) if it is deducible from the empty set.

A set \(F\) of formulas is consistent if there are at least a classical formula and at least a probabilistic formula that are not deducible from \(F\).
$LPP_2 \ (4)$
\( \text{LPP}_{2}^{\text{Fr}(n)} \) vs \( \text{LPP}_{2} \)

- \( \text{LPP}_{2}^{\text{Fr}(n)} \)-Axiom

\[
\bigvee_{k=0}^{n} P = \frac{k}{n} \alpha
\]

i.e., \( \mu([\alpha]) \in \{0, \frac{1}{n}, \ldots, \frac{n-1}{n}, 1\} \)

- instead of \( \text{LPP}_{2} \)-Rule:

  From

  \[
  \{A \rightarrow P_{\geq s} - \frac{1}{k} \alpha, \text{ for } k \geq \frac{1}{s}\}
  \]

  infer

  \[
  A \rightarrow P_{\geq s} \alpha
  \]
$LPP_{2,P,Q,O}$

Extension of $LPP_2$:

1. $O$ - recursive family of recursive subsets of $[0, 1]_Q$
2. $Q_F$, $F \in O$
3. $M \models Q_F p$ iff $\mu([p]) \in F$
Extension of $LPP_2$:

- $O$ - recursive family of recursive subsets of $[0, 1]_Q$
- $Q_F, F \in O$
- $M \models Q_F p$ iff $\mu([p]) \in F$

- $Q_F$’s and $P_{\geq s}$’s are mutually undefinable
\[ LPP_{2}^{Fr(n)}, \mu : H \rightarrow \{0, \frac{1}{n}, \ldots, \frac{n-1}{n}, 1\}, P \geq s \]

\[ LPP_{2}, \mu : H \rightarrow [0, 1], P \geq s \]

\[ LPP_{2,P,Q,O}, \mu : H \rightarrow [0, 1], P \geq s, Q_{F} \]
$LPP_{Fr(n)}^2, \mu : H \rightarrow \{0, \frac{1}{n}, \ldots, \frac{n-1}{n}, 1\}, \mathcal{P} \geq_s$

same language

diff. models

$LPP_2, \mu : H \rightarrow [0, 1], \mathcal{P} \geq_s$

$LPP_{2,P,Q,O}, \mu : H \rightarrow [0, 1], \mathcal{P} \geq_s, Q_F$
\( LPP_{2}^{Fr(n)}, \mu : H \rightarrow \{0, \frac{1}{n}, \ldots, \frac{n-1}{n}, 1\}, P \geq s \)

same language

diff. models

\( LPP_{2}, \mu : H \rightarrow [0, 1], P \geq s \)

same models
different languages

\( LPP_{2,P,Q,O}, \mu : H \rightarrow [0, 1], P \geq s, Q_F \)
\( LPP_{2}^{Fr(n)} \), \( \mu : H \rightarrow \{0, \frac{1}{n}, \ldots, \frac{n-1}{n}, 1\} \), \( P \geq s \)

same language

diff. models

\( LPP_{2} \), \( \mu : H \rightarrow [0, 1], P \geq s \)

same models

different languages

\( LPP_{2,P,Q,O} \), \( \mu : H \rightarrow [0, 1], P \geq s, Q_{F} \)

- (soundness) If \( T \vdash \phi \), then \( T \models \phi \);
- (deduction theorem) \( T \vdash \phi \rightarrow \psi \) iff \( T, \phi \vdash \psi \);
- (strong completeness) Every consistent theory is satisfiable;
- decidability: \( LPP_{2}^{Fr(n)}, LPP_{2} \)
- (un)decidability: \( LPP_{2,P,Q,O} \)-logic is undecidable.
Hierarchies: $LPP_2$ and $LPP_{2,P,Q,O}$ (1)

- Measurable models: every $[\alpha] = \{ w \in W : w \models \alpha \} \in H$

- $M(\phi)$ is the set of all $M \in M$ such that $M \models \phi$. 
Hierarchies: $LPP_2$ and $LPP_{2,P,Q,O}$ (2)

- $F_1 = \left\{ \frac{1}{2^i} : i = k, k+1, \ldots \right\}$, $k > 0$
- $F_2 = \left\{ \frac{1}{2^i} : i = 1, 2, \ldots \right\}$
Hierarchies of Probabilistic Logics

Hierarchies: $LPP_2$ and $LPP_{2,P,Q,O}$ (2)

- $F_1 = \{ \frac{1}{2^i} : i = k, k + 1, \ldots \}, \ k > 0$
- $F_2 = \{ \frac{1}{2^i} : i = 1, 2, \ldots \}$

- $F_1 = F_2 \cap [0, \frac{1}{2^k}]$

- $\mathcal{M}(Q_{F_1} \alpha) = \mathcal{M}(Q_{F_2} \alpha \land P \leq \frac{1}{2^k} \alpha)$
Hierarchies: $LPP_2$ and $LPP_{2,P,Q,O}$ (3)

$F \subseteq [0, 1]_Q$

quasi complement: $1 - F = \{1 - s : s \in F\}$
Definition

$O_1$ is **representable** in $O_2$ if every $F_1 \in O_1$ can be expressed as:

- a finite union of
- finite intersections of sets, differences between sets and quasi complements of
- sets from $O_2$ and $[r, s], [r, s), (r, s]$ and $(r, s)$, $r, s \in [0, 1]_\mathbb{Q}$
Hierarchies: $LPP_2$ and $LPP_{2,P,Q,O}$ (4)

**Definition**

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- a finite union of
- finite intersections of sets, differences between sets and quasi complements of
- sets from $O_2$ and $[r, s]$, $[r, s)$, $(r, s]$ and $(r, s)$, $r, s \in [0, 1]_Q$

**Definition**

$L_2$ is **more expressive than** $L_1$ if for every formula $\phi \in \text{For}(P, Q, O_1)$ there is a formula $\psi \in \text{For}(P, Q, O_2)$ such that

$$M(\phi) = M(\psi)$$
Hierarchies: $LPP_2$ and $LPP_{2,P,Q,O}$ (5)

Theorem

$O_1$ is representable in $O_2$ iff $L_2$ is more expressive than $L_1$
Hierarchies: $LPP_2$ and $LPP_{2,P,Q,O}$ (5)

Theorem

$O_1$ is representable in $O_2$ iff $L_2$ is more expressive than $L_1$

Definition

$\overline{O}$ is the family of all recursive subsets of $[0,1]_Q$ representable in $O$.

$LPP_{2,P,Q,O} \implies LPP_{2,P,Q,\overline{O}}$
Hierarchies: $LPP_2$ and $LPP_{2,P,Q,O}$ (6)

\[ \mathcal{O}^* = \{ \overline{O}_o : o \in \mathcal{O}/\sim \} \]

**Theorem**

The structure $(\mathcal{O}^*, \subseteq)$ is a non-modular non-atomic lattice, with the smallest element which is $\sigma$-incomplete and without any maximal element.

the smallest element: $LPP_2$
Hierarchies: $LPP_2^{Fr(n)}$ (1)

- $LPP_2^{Fr(2)}$, $\mu : H \rightarrow \{0, \frac{1}{2}, 1\}$
- $LPP_2^{Fr(4)}$, $\mu : H \rightarrow \{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\}$
Hierarchies: \( LPP^\text{Fr}(n)_2 \) (1)

- \( LPP^\text{Fr}(2)_2 \), \( \mu : H \rightarrow \{0, \frac{1}{2}, 1\} \)
- \( LPP^\text{Fr}(4)_2 \), \( \mu : H \rightarrow \{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\} \)

\[
\models_{LPP^\text{Fr}(2)_2} P > \frac{1}{2} \quad p \rightarrow P \geq 1 \ p
\]

\[
\not\models_{LPP^\text{Fr}(4)_2} P > \frac{1}{2} \quad p \rightarrow P \geq 1 \ p
\]
Hierarchies of Probabilistic Logics

Hierarchies: $\mathbb{LPP}^\mathbb{Fr}(n)_2$ (1)

- $\mathbb{LPP}^\mathbb{Fr}(2)_2$, $\mu : H \rightarrow \{0, \frac{1}{2}, 1\}$
- $\mathbb{LPP}^\mathbb{Fr}(4)_2$, $\mu : H \rightarrow \{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\}$

- $\models_{\mathbb{LPP}^\mathbb{Fr}(2)_2} P > \frac{1}{2} \Rightarrow P \geq \frac{1}{2} \Rightarrow p$
- $\not\models_{\mathbb{LPP}^\mathbb{Fr}(4)_2} P > \frac{1}{2} \Rightarrow P \geq \frac{1}{2} \Rightarrow p$

- $P > \frac{1}{2} \Rightarrow p \Rightarrow P \geq \frac{1}{2} \Rightarrow p \land \bigwedge_{\alpha \in \text{Forc}} (P = 0 \alpha \lor P = \frac{1}{2} \alpha \lor P = 1 \alpha)$
Hierarchies: $LPP_2^{Fr(n)}$ (2)

Theorem

$LPP_2^{Fr(n_2)}$ is more expressive than $LPP_2^{Fr(n_1)}$ iff $Fr(n_1) \subseteq Fr(n_2)$

Theorem

The hierarchy is atomic and non-modular lattice with minimum and without a maximal element.

the smallest element: $LPP_2^{Fr(1)}$, $\mu : H \rightarrow \{0, 1\}$
Two hierarchies

\[ P_{\geq \frac{1}{2} p} \rightarrow P_{\geq 1 p} \land \bigwedge_{\alpha \in \text{ForC}} Q_{\{0, \frac{1}{2}, 1\}}^{\alpha} \]
Two hierarchies

\[ P_{\geq \frac{1}{2}} \] \implies P_{\geq 1} \land \bigwedge_{\alpha \in \text{Forc}} Q_{\{0, \frac{1}{2}, 1\}} \alpha\]
Answers to Lluis’ and Henri’s questions (1)

1. L & H: *What graded notion(s) are you handling?*
   - We use probabilities to quantitatively model uncertain beliefs.

2. L & H: *What kind of ”weighted” logic are you developing?*
   - We develop probability logics with probability modalities.
1. L & H: *What graded notion(s) are you handling?*
   - We use probabilities to quantitatively model uncertain beliefs.

2. L & H: *What kind of ”weighted” logic are you developing?*
   - We develop probability logics with probability modalities.
   - values of probability functions in non-Archimedean structures
   - intuitionistic logic, temporal logic, …
   - conditional probabilities, first order logic
3. L & H: *For what purpose?*

- checking consistency of finite sets of rules in expert systems
- deducing probabilities of conclusions from uncertain premisses
- modelling non-monotonic reasoning, spatial-temporal-uncertain reasoning
- modelling situations when classical reasoning is not adequate (intuitionistic logic)
List of related publications

http://www.mi.sanu.ac.rs/~zorano/papers.html