

An Approach to Interaction-Based Concept Convergence in Multi-Agent Systems

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Abstract. The problems that are faced by two communicating systems that have different knowledge representations can be difficult to formalize due to the different notations of these different knowledge representation approaches. We present a new formalism that has been conceived to describe how communicating systems can argue the meaning of their concepts. This aims to help to develop new methods for the systems with heterogeneous vocabularies to reach mutual understanding.

Keywords. Multi-Agent Systems, Knowledge-Sharing, Formalism, Interaction-Based, Semiotics

1. Introduction

“What ya going to have, Mac? Something to eat?”

“Yeah. What kind of sandwiches ya got besides hamburgers and hot dogs?”

“How about a ham’n cheese sandwich?”

“Nah ...I guess I’ll take a hamburger again.”

“Hey, that’s no hamburger; that’s a cheeseburger!”

Charles O. Frakes, 1962 [7]

The first requirement for a communication can be seen as the ability for two systems to send and receive signals. If two systems want to communicate, we assume that they will need more than sending and receiving random noises. This is where the key notion of *meaning* appears. Our two systems will need of course to exchange symbols, but also to *agree* that these symbols have – if not equal, at least compatible – meanings. A global agreement on the symbol-meaning associations from both systems leads to mutual intelligibility, while disagreements might cause you to eat something different than what you thought you ordered. How to reach such an agreement is the object of various fields of research that we can refer to by the term *concept convergence*.

1.1. Context

Before addressing any issue in communicating systems, there should be a clear definition for each of the three key elements listed above: the symbol, the meaning and the nature of their relation.

A definition for the term *symbol* is given relatively to the notion of sign by Pierce: there are three kinds of sign – icons, indexes and symbols. A symbol is a sign that “denotes its object solely by virtue of the fact that it will be interpreted to do so”: it possesses an abstract relation with its associated meanings, (for instance the words “poule”, “pollo” and “chicken” do not share any physical connection with the animal that they represent). Symbols are the category of sign that we use in computer science.

Conversely, the term *meaning* is more tricky to define. There is a long story of debate within the field of philosophy of language over the meaning of *meaning*. However, Goldstone [8] simplifies them into two major branches: the *conceptual web* approach and the *externally grounded concepts* approach. The former approach considers that the meaning of a concept is given by its connections to other concepts [3] [6] [11] in a similar way as the early semiotic theories of Ferdinand De Saussure. In contrast, the last approach considers that a meaning has to be *grounded*. The term has been coined by Steven Harnard and its thought experiment of the Chinese room [9]. The conceptual web approach finds its foundation in the British empiricist movement and continues to influence science today, through the embodied cognition theory in cognitive sciences for instance.

With the diversity of approaches to give a definition of *meaning*, and often in contradiction with each other, it can be challenging for researchers to address formally an issue in communicating systems, especially if the research takes place in an interdisciplinary context that combine the two approaches. We do not intend to list the advantages and drawbacks of each approach, we rather intend to create a formalism that can be used in both.

1.2. Aim of our Formalism

While this formalism does not define the notion of meaning, it defines a set of other related notions that can help anyone to create a formalization of its custom notion of meaning – regardless to where this notion falls in the spectrum of possible approaches, between the conceptual web and the externally grounded concepts. Moreover, even if a scientific community decides to use a huge set of definitions for the notion of meaning, as long as they notions are defined in the same formalism it will be possible to compare them, and their related work.

Researchers from many fields including (but not limited to) Machine Learning, Ontology Matching or Multi-Agent Systems need to address a broad range of situations where two communicating systems, that we will refer to as *agents* from now on, do not agree on the meaning of their symbols. In order to clarify their problem and solution, they need a robust and domain independent formalism that provides clear definitions for notions including *meaning*, *agreement on the meaning*, *concept* etc.

We provide a formalism that defines these notions in a compatible way with respect to each other. While this formalism has been initially created in the context of symbolic inductive learning [2], it incorporates elements from semiotics and can be used with a large set of approaches. The formalism’s inherent flexibility comes from three main points:

The formalism is designed for an interactive approach In most of ontological approaches, a third system – the oracle – will solve the disagreements on the meaning between two other agents by accessing the combined knowledge of both agents [5]. We designed a formalism that is able to work in an interactive way, where the two disagreeing agents are able to solve their disagreements by interacting with each other, without any help from a third part. While this allows the formalism to remain compatible with classical approaches of ontology matching, the growing interest from the research community in interactive approaches [4] allows this point to be a main feature of our formalism.

The formalism is designed for a lazy approach This formalism is designed to allow a lazy approach of resolving disagreements on the meaning: a disagreement can be solved *on demand* when it arises, without requiring an exhaustive analysis of both agent’s symbols and meanings. For this reason, the formalism can represent a concept as itself, not only through all its relations with other concepts (external concepts approach). The exhaustive approach usually found in ontology matching (conceptual web) can also be formalized, due to the notion of contrast set which defines a set of concepts by their respective relations.

The formalism differentiates symbols and meanings The formalism establishes a strict distinction between the symbol and what it stands for, allowing the agents to clarify the notions of *symbol*, *meaning*, and these two notions’ relations. This allows researchers to formalize models where the agents can dynamically modify the symbol of a concept without modifying the relations that the concept has with other concepts, in the case of a conceptual web approach.

2. Semiotic Formalism

2.1. Presentation

The formalism uses the three basic components of semiotics from the semiotic triangle of Odgen and Richards [10], and draw a correspondence between the triangle’s symbol, thought of reference and referent with respectively the sign, the generalizations and the examples more commonly used in machine learning (see Figure 1).

These three components are assembled into a concept, and concepts are assembled among containers. There are two kinds of containers: the *contrast sets*, that are used for communication by agents, and the *hypotheses*, that are not suitable for communication but useful during the process of building new contrast sets.

2.2. Semiotic Elements

The semiotic element are the components of concepts and containers. The different elements that an agent has perceived in its environment are called *examples* – real world observations of objects. For example, a specific chair from an office is an example. They are noted e_i . A context $E = \{e_1 \dots e_n\}$ is a set of examples. An extensional definition on a context is a set of examples $E_i \subset E$.

The agents are using ψ -terms as their representation language. An agent represents an example e_i using a feature-term. A generalization g_j of a set of examples E_i is an other

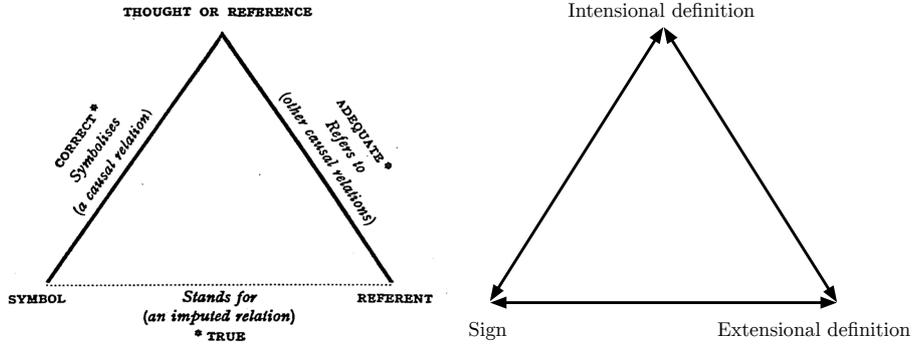


Figure 1. The semiotic triangle representations of a concept to compare the original semiotic elements from Odgen and Richard (left) to the ones used in our formalism (right).

feature-term that verifies $\forall e_i \in E_i, g_j \sqsubseteq e_i$. An intensional definition $I_i = \{g_1 \dots g_i\}$ is a set of generalizations.

A sign s_i is an abstract entity that exists only in the communication between two agents. A concept $C_i = (s_i, I_i, E_i)$ is the triadic relation between a sign¹, an intensional definition and an extentional definition. The relation should verify: $\forall e_i \in E_i, \exists g_i \in I_i$ as $g_i \sqsubseteq e_i$. If the concept C_i belongs to an Agent A_k , we note it C_i^k .

2.3. Containers

Containers are a dyadic relation between a context E and a set of concepts $\{C_1, \dots, C_n\}$.

A hypothesis $H = (E, \{C_1, \dots, C_n\})$ is a container such that the set of examples $E_1 \cup \dots \cup E_n \subseteq E$ and where the signs of the concepts are different: $\forall C_i, C_j \in \{C_1, \dots, C_n\}, i \neq j \rightarrow s_i \neq s_j$. We will note the context of the hypothesis E_H .

A contrast set $K = \{C_1, \dots, C_n\}$ is a container such that the context $E = E_1 \cup \dots \cup E_n$, the concepts are disjoint $\forall i, j, E_i \cap E_j = \emptyset$ and the signs of the concepts are different. K is a partition of E . We will note the context of the contrast set E_K .

3. Agreement and Disagreement

In multi-agent systems, agents are using their contrast sets to encode a meaning into a sign and decode signs into meanings. Understanding the conditions to mutual intelligibility between two agents requires to study the relations between the contrast sets they are using, $A_1.K$ for agent 1 and $A_{-1}.K$ for agent 2. Since we aim to grant the agents with the ability to modify their contrast sets into $A_1.K'$ and $A_{-1}.K'$, the relation between an initial version $A_k.K$ of a contrast set and its latest version $A_k.K'$ needs to be studied as well.

¹Notice that there is no constraint on the sign, therefore the choice of a sign for a concept is arbitrary. The arbitrariness of the sign means that all of those signs are *symbols* from a semiotic point of view.

3.1. Agreement over Meaning

Agents always aim to set the contrast set they are using as intelligible for both of them: we refer at this aim as *mutual intelligibility*. Each agent is in charge of evaluating if a contrast set is intelligible for himself. The different criteria that an agent uses to evaluate whether or not a contrast set is intelligible from its point of view, are called the *agreements*.

3.1.1. Synchronic Agreement

The first criterion to the intelligibility, is to share the same signs for concepts with same extensional definitions, and different signs for concepts with different extensional definitions. In order to avoid ambiguity, the contrast sets still need to have the extensional definitions of its concepts disjoint. This criterion is the *synchronic agreement*, referring to the sub-field of linguistics that describes language at a specific point of time, by opposition to diachronic. The synchronic agreement can be formalized as $\forall(C_i^k, C_j^{-k})$ with $C_i^k \in A_k.K$ and $C_j^{-k} \in A_{-k}.K$, $(E_i^k = E_j^k) \Leftrightarrow (s_i = s_j)$, $(E_i^k \cap E_j^k = \emptyset) \Leftrightarrow (s_i \neq s_j)$ and $(E_i^k = E_j^k) \vee ((E_i^k \cap E_j^k) = \emptyset)^2$.

3.1.2. Diachronic Agreement

A second criteria reflects the expected trade-off between generalization and specialization among a contrast set. We want the agents to have concepts general enough to remain as far as possible from the extreme scenario of having one different concept by example. However at the same time, we want the agents to have concepts specific enough to remain as far as possible from the extreme scenario of having only one concept for all examples.

We consider here that agents initially start with contrast sets that were intelligible from their point-of-view. Therefore, we use there level of trade-off as the baseline for the expected generality and specialization of any new contrast set's concepts. However, if agents need to change their concepts in order to reach mutual intelligibility, they will have to make concessions either on the generality or the specialization of their concepts. Therefore, we have to make a choice between having more general or more specific concepts as the result of the changes.

Since the number of concepts among the contrast set is from far inferior to the number of examples in its context, the risk of having an over-generalized contrast set is more constraining than the risk of an over-specialized. For this reason, an agent can only give its agreement to a contrast set that is more specific than its initial one. It will be the role of the argumentation to limit the concepts' specialization. We call this criterion the *diachronic agreement*, in reference to linguistics where diachronic refers to the development and evolution of language through time. The diachronic agreement can be formalized as $\forall C_i^k \in A_k.K, !\exists(C_j^k \dots C_n^k) \in A_k.K'$ such that $E_j \cap \dots \cap E_n = \emptyset$ and $E_j \cap \dots \cap E_n = E_i$.

3.2. Disagreements over the Meaning

Within a mutli-agent system, the success of a communication lies on multiple criteria that we present as agreements. Every contradiction to these criteria are seen as *disagree-*

² \vee is an infix for "exclusive or".

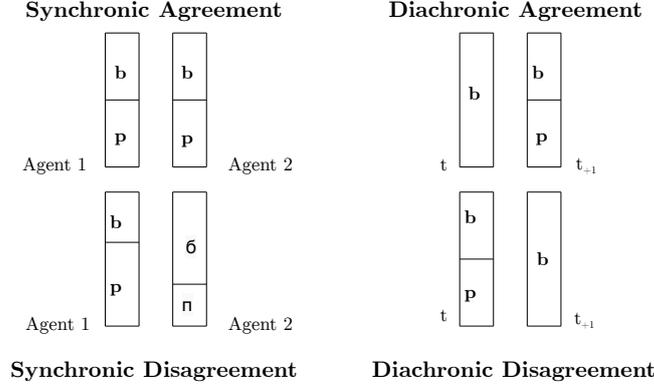


Figure 2. A synchronic (left) and diachronic (right) agreement (top) and disagreement (bottom). The synchronicity is related to the concepts' relations between two different agents' contrast sets, while the diachronicity is linked to the evolution of one same contrast set over the time.

ments over these criteria. Since the criteria over one agent's agreement are now clear, it is possible to express which relations between concepts can contradict them. Since we have synchronic and diachronic agreements, we refer to their respective contradictions as synchronic and diachronic disagreements.

First, the synchronic agreement can be contradicted by two kinds of relations: relations between two concepts' signs, or relations between two concepts' extensional definitions.

The first kind of synchronic disagreement is related to the signs. If two concepts $C_i^k \in A_k.K$ and $C_j^{-k} \in A_{-k}.K$ have different signs but their extensional definitions are the same, situation that is formalized as $(s_i = s_j) \wedge (E_i \neq E_j)$, the two concepts are in a relation of *homonymy*. However, if the two concepts have different sign but the same extensional definition, formalized as $(s_i \neq s_j) \wedge (E_i = E_j)$, the concepts are in a relation of *synonymy*.

The second kind of synchronic disagreement is independent from the signs, and lies only on extensional definitions. If one of the two concepts' extensional definition is a proper subset of the other concept's extensional definition such that $E_i \subset E_j$, then the two concepts are in a *hypo/hypernymy* relation. The other possibility is that the two extensional definitions are overlapping onto each others such that $(E_i \cap E_j \neq \emptyset) \wedge (E_i \cap \bar{E}_j \neq \emptyset) \wedge (\bar{E}_i \cap E_j \neq \emptyset)$. This relation is referred by as an *overlapping* relation.

The diachronic agreement can be contradicted in two ways. The first one is by having at least one concept from a contrast set that does not have its extensional definition included in the extensional definition of a concept from an anterior contrast set. The relation causing the disagreement is referred to as a *confusion*, and adopting the same notation as in section 3.1.2 about K and K' , we formalize it as $\exists C_i^k \in K' \wedge \nexists C_j^k \in K$ such that $E_i \subset E_j$.

The last contradiction arises when some examples from one contrast set's context is not present in a posterior contrast set's context. This particular case is not caused by a relation between concepts, but by a relation between contexts. It is noted $E_{K'} \subset E_K$, and is referred by as the *incompleteness* relation.

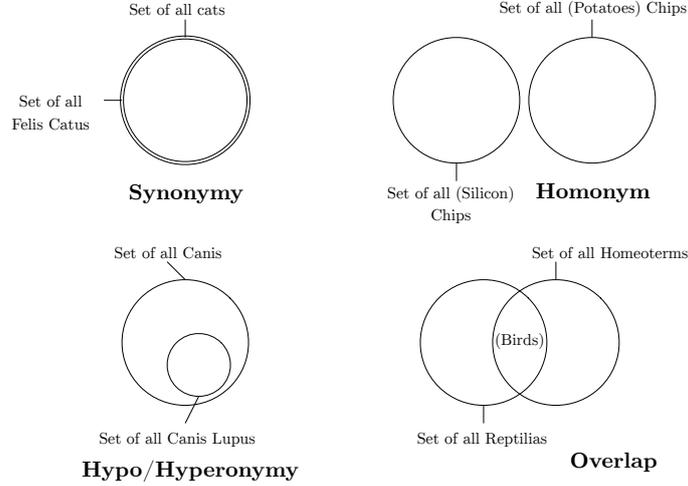


Figure 3. The different types of synchronic disagreements between two concepts. The concepts' extensional definitions are represented as Venn diagrams, using examples from zoology. The disagreement can arise from the sign of the concept (top) or only the number of examples that their extensional definitions share.

3.3. Overall Relations

The relations behind the diachronic disagreement are easily identifiable by one agent, since it possesses the two contrast sets with the incriminated relations. However, the case of the synchronic disagreement is more complicated. Since the relations causing this disagreement are between examples from different agent, and that agents are not necessary having the same examples in their contexts, the agents might not individually identify a disagreement that occurs when you consider the total set of examples present in both agents. This kind of disagreements is referred to as *second order* disagreements. To detect and solve the second order disagreements with the elements of our protocol listed so far, the agents would need to transfer all their example to each other.

Fortunately, by granting the agents the ability to communicate with each other about their point-of-view on the relation between two concepts, it is possible to overcome the requirement of an extensive example transfer. To do so, an agent has to receive all the pairs of signs and intensional definitions from the other agent's contrast set. When the agent A_k received all the pairs from $A_{-k}.K$, it can build a hypothesis H with the context E_K of its contrast set and a concept $C_j^k = (s_j, I_j, E_j = \{e \in E \mid \exists g \in I_j \wedge g \sqsubseteq e\})$ for each received pair (s_j, I_j) .

After building the hypothesis, A_k can examine the relations between the concepts from its contrast set and the concepts from its hypothesis. These relations are referred to as the *local* relations. If A_k has m concepts and A_{-k} has n concepts, each agent has a set of $m * n$ local relations. In order to standardize the communication, we allow the agents to express their relations through four different symbols. A relation r between the concepts C_i and C_j takes its value from the set $\{\equiv, \circ, \odot, \otimes\}$ depending on the relation between the concepts' extensional definitions E_i and E_j as follow:

- $r = \equiv$ if E_i is equivalent to E_j .

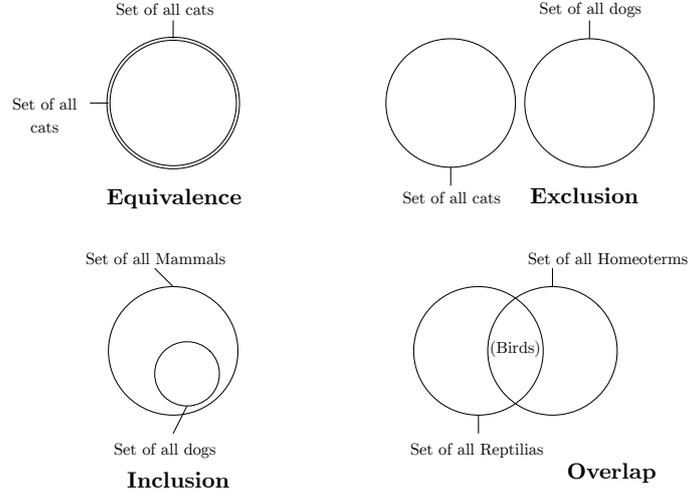


Figure 4. An illustration of the different possible relations between concepts with examples from zoology. A concept is always equivalent to itself, and have different relations according to the number of examples that it shares with another concept.

- $r = \circlearrowleft$ if E_i and E_j are disjoint.
- $r = \odot$ if $E_i \subset E_j$ or $E_j \subset E_i$.
- $r = \otimes$ if $E_i \cap E_j \neq \emptyset$, $E_i \cap \bar{E}_j \neq \emptyset$ and $\bar{E}_i \cap E_j \neq \emptyset$.

A_k knows the local relation between each pair of concepts ($C_i^k \in A_k.K$, $C_j^k \in A_k.H$). Since A_{-k} behave similarly, A_k knows that there exists two concepts $C_i^{-k} \in A_{-k}.H$ and $C_j^{-k} \in A_{-k}.K$. By getting the local relation between C_i^{-k} and C_j^{-k} , A_k can infer the *overall* relation between C_i^k and C_j^k according to the following rule:

- **case 0:** if the two local relations are the same, the overall relation is also the same.
- **case 1:** if the local relations are $C_i^k \odot C_j^k$ and $C_i^{-k} \circlearrowleft C_j^{-k}$, the overall relation is $C_i^k \otimes C_j^k$.
- **case 2:** if the local relations are $C_i^k \odot C_j^k$ and $C_i^{-k} \equiv C_j^{-k}$, the overall relation is $C_i^k \odot C_j^k$.
- **case 3:** if the local relations are $C_i^k \equiv C_j^k$ and $C_i^{-k} \circlearrowleft C_j^{-k}$, the overall relation is $C_i^k \otimes C_j^k$.
- **case 4:** if the local relations are $C_i^k \otimes C_j^k$ and $C_i^{-k} \odot C_j^{-k}$, $C_i^{-k} \circlearrowleft C_j^{-k}$ or $C_i^{-k} \equiv C_j^{-k}$, the overall relation is $C_i^k \otimes C_j^k$.

Since these relations are symmetrical, this set of rules covers any possible combination of relations. The overall relation is the relation that the two concepts C_i^k and C_j^k would have in an overall context $E = E_{A_k.K} \cup E_{A_{-k}.K}$.

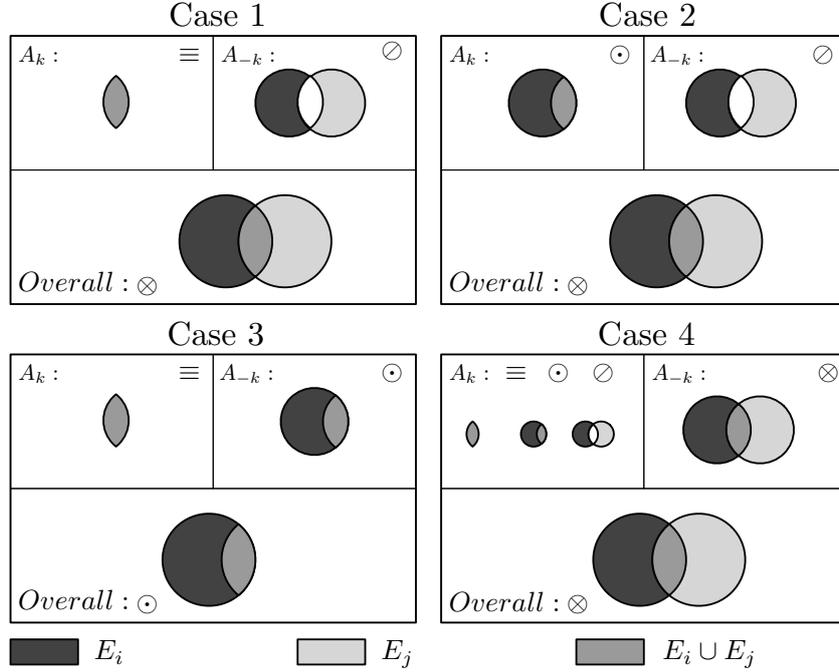


Figure 5. The four cases where the two local relations are not matching, and their associated overall relations.

3.4. Hierarchy

The overall relation does not provide enough information yet to decide an agreement in all cases. The \odot relation does not specify which concept includes the other in the overall relation. In order to resolve this ambiguity, the agents have to decide on a *hierarchy* between the concepts.

If we look at the vocabulary from section 3.2, we see that the case of a concept's extensional definition having the extensional definition of another concept as a subset is referred to as a *hypo/hypernymy*. This name comes from the field of linguistic: a hyponym shares a *type-of* relation with its hypernym, where the most general concept is the hypernym and the most specific is the hyponym. An example is the color *navy-blue*, that is a hyponym of *blue*. Deciding a hierarchy, in our case, means deciding which of the two concepts of the overall relation is the hyponym, and which one is the hypernym.

The hierarchy among the overall relation is deciding according to the following rules. There are five possibilities that leads to the relation \odot as the overall relation between two concepts C_i^k and C_j^{-k} . In three of these possibilities, both local relations were already a \odot relation:

- if $E_i \subset E_j$ and $E_j \subset E_i$, or $E_j \subset E_i'$ and $E_i \subset E_j$, the overall relation is in fact $C_i^k \otimes C_j^{-k}$.
- if $E_j' \subset E_i$ and $E_j \subset E_i$, C_j^{-k} is the hyponym and C_i^k the hypernym.
- if $E_i \subset E_j$ and $E_i \subset E_j$, C_i^k is the hyponym and C_j^{-k} the hypernym.

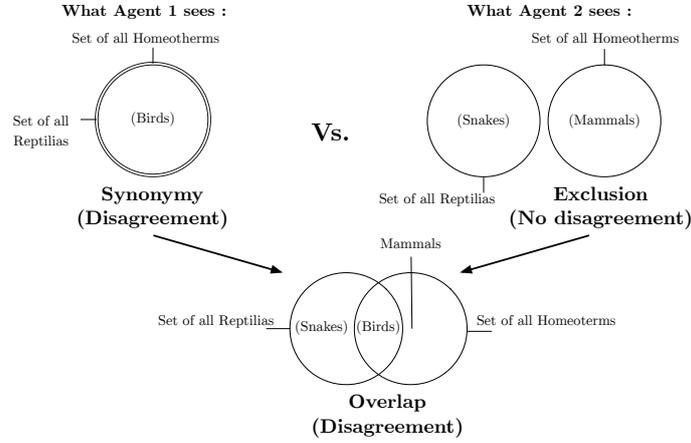


Figure 6. In some situations, the relation between two concepts might be seen differently depending on which agent observes it. This is due to the different contexts of their contrast set. In the Figure, the two agent are seeing the relation between two concepts as *equivalence* and *disjoint* (top) while the reality is a third relation of *overlap* (bottom)

In the last two possibilities, one of the local relation was not \odot but \equiv , as in case 2 of section 3.3:

- if $E_i \subset E_j$ and $E_i \equiv E_j$, or $E_i \equiv E_j$ and $E_i \subset E_j$, C_i^k is the hyponym and C_j^{-k} the hypernym.
- if $E_j \subset E_i$ and $E_j \equiv E_i$, or $E_j \equiv E_i$ and $E_j \subset E_i$, C_j^{-k} is the hyponym and C_i^k the hypernym.

3.5. Complements

These additions are not part of the core formalism, but can be still useful for researchers that face particular situations.

Ambiguity of the Sign: Since sometimes the concepts C_i^k from $A_k.K$ and C_j^{-k} from $A_{-k}.K$ will be in a situation where $i = j$. In this situation, the concept C_i from A_k and the concept C_j from A_{-k} can be both noted C_i^k or C_j^k . In order to remove this ambiguity, we will note C_j^k the concept that belongs to $A_k.H$. This way, the apostrophe marks the belonging to a hypothesis. In the situation where $i \neq j$, the absence of ambiguity allows us to not put the apostrophe.

Co-hyponymy: In this Section, four notions from semiotics have been introduced to describe the *semantic* relation³ between two concepts. These notions are: hyponymy, hypernymy, synonymy and homonymy. We add now a fifth notion, the co-hyponymy. A set of concepts C_{h_1}, \dots, C_{h_n} are co-hyponyms of a common hypernym C_H if the co-

³We are not referring there to the local and overall relations, which do not belong to the field of semantic but only to the field of set theory.

hyponyms' extensional definitions E_{h_1}, \dots, E_{h_n} are a partition of the extensional definition of the hypernym's extensional definition E_H .

4. Discussion

The formalism is versatile and can be used in many of the fields cited in the introduction (Ontology Matching, Symbolic Inductive Learning, Semiotic ...). However, the real interest of the formalism lies in its compatibility with works that belong to a *combination* of these fields. The main feature of the formalism is its ability to formalize concepts of our language that we use to talk about language itself. This function of language, the *reflexive* function, can be confusing sometimes. Expressions like *meaning of the meaning*, or *concept of concept* are often useful to use in a research paper from one of the previously cited field, but confuses the reader. This problem is exacerbated in the case of interdisciplinary texts, where making a reference to which instance of a notion like concept is used may degrade the clarity of the text even more. By formalizing each of these terms together with their relations, we aim to improve the clarity of interdisciplinary research reports: for instance C refers to the formal notion of a concept while *concept* refers to the notion of concept in its broad meaning. The *concept of concept* is now noted $C_{concept}$.

We do not provide a definition for the meaning. Since this notion is subjective and defined in different ways among different approaches, we prefer to let each user of the formalism choose the way that he/she considers the most appropriate to define the notion of *meaning*. Different ways to formalize the meaning of a concept C_i include (but are not limited to):

- its intensional definition I_i (dualism)
- its extensional definition E_i (materialism)
- the concept C_i itself, as the tuple $\{s_i, I_i, E_i\}$ (externally grounded concepts)
- the relation r of the concept C_i with another concept C_j (conceptual web)

The formalism has been used in an experiment where a pair of agents resolves simple scenarios of disagreements by arguing over the meaning of their concepts [1].

5. Conclusion

We presented an original formalism that can fit, but is not limited to, interdisciplinary approaches of communication. This formalism presents the basic elements *sign*, *extensional definition* and *intensional definition*, how their interaction forms a concept, how concepts are organized to partition a linguistic context, and finally how these organizations can cause disagreements on the meaning, what is the typology of these disagreements and how we can evaluate a good organization that results in an agreement, even from the point of view of an agent that has only partial information on the linguistic context of its interlocutor. We are looking forward to see how the research community will use this formalism to disambiguate abstract notions of our language.

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